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Newton's Law Modifications from Extra Dimensions.

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Abstract

We discuss in brief the corrections to Newton's inverse square law due to possible existence of extra decompactified dimensions. Experimental bounds on the parameters (compactification radius and the number of extra dimensions) are also given .

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1. Introduction

One of the most important issues in particle physics today is the hierarchy problem, in other words, the vast difference of electroweak and the unification scales. Indeed, according to the traditional approaches, like Grand Unification and Supersymmetry, the unification of the fundamental forces occurs at a scale M_U which is close to the Plank mass M_{Pl}. This is some 15 orders of magnitude larger than the electroweak scale m_W, not testable by any earth-based experiment.

In early 90's it was realized that extra decompactified dimensions may open up at a scale as low as a few TeV[1]. Based on this idea, a new proposal for solving the hierarchy problem was introduced[2, 3] which brings the fundamental Plank scale close to the electroweak scale. In the last two years there has been intensive study of gravitational models which view our world as a 3-brane immersed in a higher dimensional [1] space [6, 7]. The success of these scenarios is based on the fact that the Standard Model (SM) particles can be confined on the 3-brane. This is not possible for the gravity however, which propagates in all dimensions, as it is the dynamics of the space-time itself. In figure 1 we see the simplified picture of these scenarios: The thick vertical line stands for the three-brane (our world) which can be seen as the projection of a two-dimensional surface with one dimension (depth) representing the 3 spatial dimensions and the other dimension (height) representing the time component. The extra dimensions are transverse to the 3-brane. The 3-brane may have a thickness along the extra dimension which is related to the weak scale $\Delta y \sim m_W^{-1}$. (Other lecturers will further explain how this thickness may arise dynamically [9, 10].) Spin 0,1 and 1/2 fields are trapped on the brane, while those states with spin higher than 1 (the graviton with spin 2 and the gravitino with spin 3/2) are in principle free to travel in the higher dimensional bulk.

[1] A similar idea which views our world localized on a defect of a higher dimensional space has been considered in [4],[5].

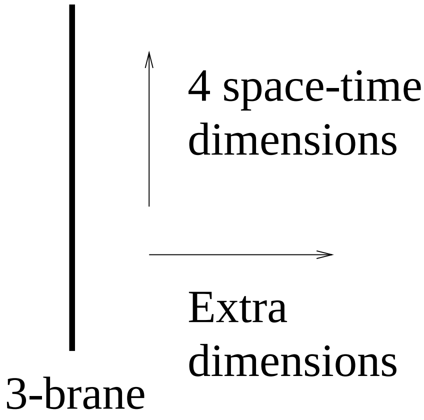

Figure 1: Our world as a 3-brane immersed in a higher dimensional space.

One of the main experimental implications of the extra dimensions is the modification of Newton's inverse-square law. Large extra dimensions however, lead to unacceptable modifications. Therefore, in order to avoid hard violations of the gravitational laws in a higher dimensional space, two possible solutions were proposed:
1) If the 4d-space-time and the extra dimensions form a direct product (factorizable geometry of space-time),

$$d\,s^2 \;=\; g_{\mu\nu}(x)dx^\mu dx^\nu + \mathcal{M}_n(y_i)dy_i^2 \tag{1}$$

then any additional dimension must be compact [2, 3]. As we will show, the extra-dimension(s) compactification radius must be very small (in the sub-millimeter range) in order to comply with the experimental verifications of Newton's law.

2) According to another proposal [6], if the enhanced space is not factorizable, then it is possible to live in $(4+n)$ non-compact dimensions. In references [6, 7] our 4d-world is extended to a 5d space-time with a *warped* metric of the form

$$d\,s^2 \;=\; g_{\mu\nu}(y)dx^\mu dx^\nu + dy^2 \tag{2}$$

Here the 4d space-time metric $g_{\mu\nu}$ depends exponentially on the extra dimension y

$$g_{\mu\nu}(y) = e^{-k|y|}\eta_{\mu\nu}, \quad k \sim M_{Pl} \tag{3}$$

The exponential suppression is called '*warp*' factor and suppresses all masses of the particles living on the 3-brane.

In this talk, I will discuss corrections to the Newton's law implied by the above scenarios as they have been derived in [11, 12]. Further, I will derive the constraints they imply on the number and size of extra dimensions in experimental observations [13].

2. Compact Extra Dimensions

For simplicity, let us assume n-extra dimensions with common compactification radius R. Introduction of different R_i for any extra dimension y_i is possible but it only complicates the mathematics without giving more physics insight so I will avoid it. For distances much larger that R using Gauss's law one finds that the potential is given by

$$V_0(r) \quad = \quad \frac{1}{M_{Pl_{n+4}}^{n+2}} \frac{1}{R^n r} \tag{4}$$

Since this formula is relevant in the presence of extra dimensions, then the mass scale $M_{Pl_{n+4}}$ introduced, is the Plank mass of the $(n+4)$-dimensional theory. This result should be identified with the known gravitational law in 4 space-time dimensions

$$V_0(r) \quad = \quad \frac{1}{M_{Pl}^2} \frac{1}{r}, \; G_N = \frac{1}{M_{Pl}^2} \tag{5}$$

Comparing the two formulae we extract the relation

$$R = \frac{1}{M_{Pl_{n+4}}} \left(\frac{M_{Pl}}{M_{Pl_{n+4}}} \right)^{(2/n)} \tag{6}$$

The solution of the hierarchy problem implies that the fundamental Plank mass of the higher dimensional theory should be around the TeV scale. This implies that the radius of the compactified dimensions is

$$R \approx 1.9 \times 10^{31/n-17} \left(\frac{TeV}{M_{Pl_{n+4}}} \right)^{1+2/n} \tag{7}$$

If there is only one extra dimension then $R \sim 10^{13} cm$ which would imply corrections to the Potential at solar distances!!! Therefore we should have at least $n = 2$ which brings the decompactified radii at the sub-millimeter range, i.e. just below the present sensitivity of gravitational experiments. From the above, it is obviously extremely interesting to study [11, 12] the type of corrections to Newton's Law in particular close to the vicinity of the compactification radius.

I will assume toroidal compactification of common radius R. Then the differential equation governing the behaviour of the gravitational potential is written

$$\nabla^2 V(r) = -\frac{1}{R^n}\delta^3(\vec{x}-\vec{y})\delta^n(\vec{\theta}-\vec{\theta}_0) \tag{8}$$

where $\vec{x}, \vec{y}$ are vectors of the three dimensional space while $\vec{\theta} = (\theta_1, \ldots \theta_n)$. The solution involves an integral of a product of *theta*-functions which after integration results to

$$V(r) = \frac{1}{(2\pi R)^n}\frac{1}{r}\left(1 + 2\sum_{\vec{m}} e^{-|m|r/R}\cos(\vec{m}\cdot\vec{\theta})\right) \tag{9}$$

where the summation is over the n infinite towers of Kaluza-Klein (KK) excitations, $\vec{m} = (m_1, \ldots, m_n)$.

For distances $r > R$ we may keep only the leading term of the series which gives an exponential attractive correction to the gravitational potential of the form

$$V_{tor}(r)|_{r>R} \approx V_0\left(1 + 2ne^{-r/R}\right). \tag{10}$$

where the formula (4) has been used.

Experimental tests on the validity of Newton's law are based on corrections of the same form

$$V(r) = \frac{1}{M_{Pl}^2}\frac{1}{r}\left(1 + \alpha e^{-r/\lambda}\right) \tag{11}$$

The $\alpha - \lambda$ parameters are constrained by a number of various experiments [13, 14]. From the $\alpha - \lambda$ plot presented in figure 2 (see ref [13] for details) one sees that the constraint on

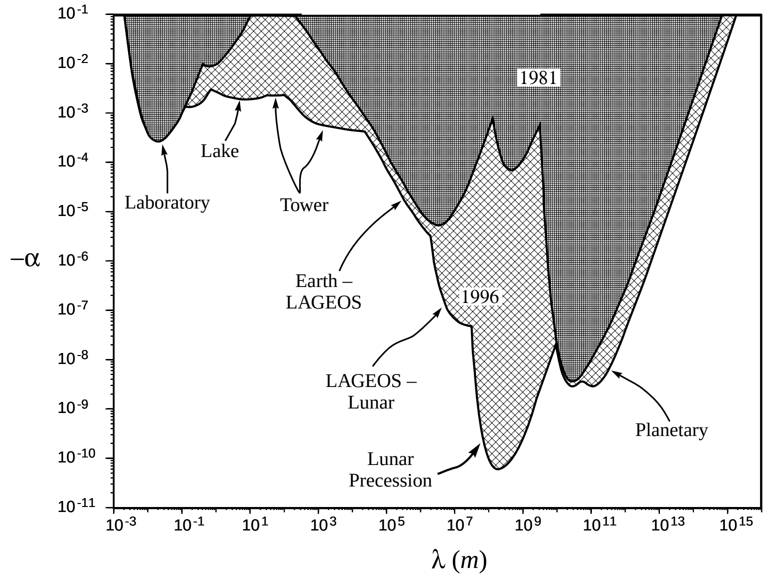

Figure 2: Various constraints on the parameters $\alpha - \lambda$ entering the corrections to Newton's Law. In the case of toroidal compactification of the extra dimensions scenaria, $\alpha = 2n$, and $\lambda = R$, (n is the number of extra dimensions, R the compactification radius.)

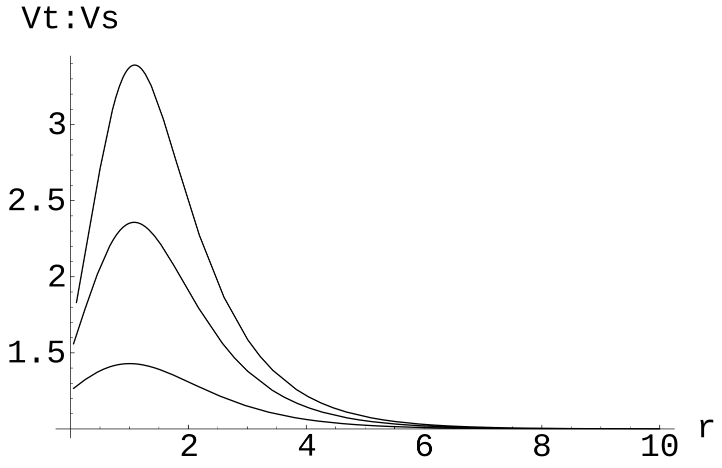

Figure 3: The ratio of the two corrected potentials as a function of the distance r. For $n = 2$ (lower curve). $n = 4$ (middle) and $n = 6$ (higher curve). (n is the number of extra dimensions.)

the parameter α is very stringent. Indeed, in most of the region one gets $\alpha \ll 1$, however for $\lambda < 10^{-3}m$ they fall off exponentially. In the case of toroidal compactification comparing (11) with (10) we identify $\lambda = R$ and $\alpha = 2n$. For two compact dimensions we may easily see that the compactification radius is of the order $\leq 1mm$. Recently, more stringent experimental short-range experiments have further improved previous constraints [14].

Similar conclusions may be reached if one assumes compactifications on a sphere. The corresponding expression for the corrected potential is then given [11]

$$V_{sph} \approx V_0 \left(1 + (n+1)e^{-\sqrt{n}r/R}\right) \tag{12}$$

In order to compare the two different type of corrected potentials we plot the ratio V_{tor}/V_{sph} in figure (3) for three choices of the number extra dimensions, $n = 2, 4, 6$. The corrections in toroidal compactification are larger mainly in the region of interest ($r \sim 1mm$), while they are enhanced for higher number of extra dimensions

3. Warped space

Let me now briefly refer to the corrections obtained in the second interesting possibility [6] of the non-factorizable higher dimensional space-time [2] . As I referred to the introduction of my talk, in this scenario the extra dimensions need not necessarily be compact. The space-time is characterized by a non-factorizable metric of the form (2) which constitutes a static solution to Einstein's equations with negative bulk cosmological constant $\Lambda < 0$. In the minimal version of this solution the space is 5-dimensional and there are two 3-branes which are located at two different points along the extra dimension. The next speaker [9] will tell you how this scenario is generalized to more than two branes. The SM particles are confined on one of the three branes with masses suppressed by the exponential factor $e^{-k|y|}$. Here $k \sim M_{Pl}$, thus due to the exponential form of the suppression factor the 'compactification scale' which corresponds to the inverse distance $M_C \sim 1/y$ need not be much smaller that M_{Pl}. It suffices to take $M_{Pl}/M_C \sim 50$ to obtain a mass scale of order m_W on our world.

What about gravity? To answer the question, one has to check the linearized graviton fluctuations around the background

$$d\,s^2 = (e^{-k|y|}\eta_{\mu\nu} + h_{\mu\nu})d\,x^\mu d\,x^\nu + d\,y^2 \tag{13}$$

Fixing the gauge so that all components are transverse and traceless, $\partial h_{\mu\nu} = 0$, $h^\mu_\mu = 0$ one finds that $h_{\mu\nu}$ obey the equation

$$\frac{1}{\sqrt{-g}}\partial_\mu(\eta^{\mu\nu}\sqrt{-g}\partial_\nu)h_{\rho\sigma} = 0 \tag{14}$$

Decomposing into a superposition of modes $h(x,y) = e^{\imath p\cdot x}\psi(y)$ and changing variable $kz \propto e^{k|y|} - 1$, we can write the differential equation as a non-relativistic Schrödinger equation

$$\left(-\frac{1}{2}\partial_z^2 + V(z)\right)\lambda(z) = m^2\lambda(z) \tag{15}$$

[2]For a recent review with a complete list of references see [8]

with $\lambda(z) = e^{k|y|/2}\Psi(y), m^2 = -p^2$ and an analogue of "non-relativistic" potential

$$V(z) = -\frac{3k}{2}\delta(z) + \frac{15k^2}{8(k|z|+1)^2} \tag{16}$$

This is a "volcano"-type potential [6] whose δ-function is responsible for the trapping of the massless graviton mode, $m = 0$, on the 3-brane. What about the $m \neq 0$ graviton modes? We see that as $z \to 0$, $V(z) \to$ constant, thus there is no mass gap, i.e. the massive spectrum is continuous. The general solution of the above Schrödinger equation gives

$$h_m(y) = N_m \left(J_2(\frac{m}{k}e^{k|y|}) + c_m Y_2(\frac{m}{k}e^{k|y|}) \right) \tag{17}$$

For $y = 0$, they become $h_m(y = 0) = \sqrt{\frac{m}{k}}$. Having found the wavefunctions of the zero mode and the KK excitations of the graviton spectrum, one can compute the gravitational potential between two particles on the brane which is proportional to [3]

$$\begin{aligned} V(r) &\propto -G_N \left(\frac{1}{r} + \int_0^\infty dm |h_m(0)|^2 \frac{e^{-mr}}{r} \right) \\ &= G_N \frac{1}{r}\left(1 + \frac{1}{k^2 r^2} \right) \end{aligned} \tag{18}$$

In contrast to the case of factorizable geometry discussed in the previous section, here the corrections exhibit a power-law behaviour.

4. Conclusions

To solve the hierarchy puzzle, two different scenarios have been proposed, one using compact and the other unconstrained extra dimensions. We find that the first scenario predicts an exponential suppression of the Newton's law corrections with the distance r. The corrections are compatible with the present day experiments which find no deviations down to submilimeter range. These scenarios solve the hierarchy problem, however they hide the arbitrariness of the low unification scale in the large compactification volume. The

[3] Further analysis maybe found in [15]

second scenario predicts a power-law behaviour correction which is negligible for distances $r > k^{-1}$, $k \sim M_{Planck}$.

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