Some Dynamical Effects of the Cosmological Constant

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Newton's law gets modified in the presence of a cosmological constant by a small repulsive term (antigarvity) that is proportional to the distance. Assuming a value of the cosmological constant consistent with the recent SnIa data ($\Lambda \simeq 10^{-52} m^{-2}$) we investigate the significance of this term on various astrophysical scales. We find that on galactic scales or smaller (less than a few tens of kpc) the dynamical effects of the vacuum energy are negligible by several orders of magnitude. On scales of 1Mpc or larger however we find that vacuum energy can significantly affect the dynamics. For example we show that the velocity data in the Local Group of galaxies correspond to galactic masses increased by 35% in the presence of vacuum energy. The effect is even more important on larger low density systems like clusters of galaxies or superclusters.

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I. INTRODUCTION

Almost two years ago two groups (the Supernova Cosmology Project [1] and the High-Z Supernova Team [2,3] presented evidence that the expansion of the universe is accelerating rather than slowing down. These supernova teams have measured the distances to cosmological supernovae by using the fact that the intrinsic luminosity of Type Ia supernovae, while not always the same, is closely correlated with their decline rate from maximum brightness, which can be independently measured. These measurements combined with redshift data for these supernovae has led to the prediction of an accelerating universe. A non-zero and positive cosmological constant Λ with

$$\Lambda \simeq 10^{-52} m^{-2} \tag{1.1}$$

could produce the required repulsive force to explain the accelerating universe phenomenon. A diverse set of other cosmological observations also compellingly suggest that the universe posesses a nonzero cosmological constant corresponding to vacuum energy density of the same order as the matter energy density [4–6].

In addition to causing an acceleration to the expansion of the universe the existence of a non-zero cosmological constant would have interesting gravitational effects on various astrophysical scales [7]. For example it would affect gravitational lensing statistics of extragalactic surveys [8], large scale velocity flows [9] and there have been some claims that even smaller systems (galactic [11] and planetary [12]) could be affected in an observable way by the presence of a cosmological constant consistent with cosmological expectations. Even though some of these claims were falsified [13–15] the scale dependence of the dynamical effects of vacuum energy remains an interesting open issue.

The effects of the cosmological constant on cosmological scales and on local dynamics can be obtained from the Einstein equations which in the presence of a nonzero cosmological constant are written as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{1.2}$$

These equations imply the Friedman equation

$$\dot{R}^2 = \frac{8\pi G}{3} R^2 \rho_M - kc^2 + \frac{\Lambda R^2}{3} \equiv \frac{8\pi G}{3} R^2 (\rho_M + \rho_\Lambda) - kc^2 \tag{1.3}$$

where the vacuum energy density ρ_Λ is defined as $\rho_\Lambda \equiv \Lambda/8\pi G$. Since the vacuum energy does not scale with redshift it is easily seen from eq. (1.3) that it can cause acceleration ($\ddot{R} > 0$) of the universe expansion.

The observational evidence for accelerating expansion along with constraints on the matter density as derived from dynamical measurements of galaxies and clusters, and additional constraints from the anisotropies of the cosmic microwave background [16], lead to consistent picture with $\frac{\rho_\Lambda}{\rho_M} \simeq 2$, with the total energy density approximately equal to the critical density necessary to solve (1.3) with k=0 ($\rho_\Lambda + \rho_M \simeq \rho_c$).

However, the vacuum energy implied from eq. (1.1) ($10^{-10} erg/cm^3$) is less by many orders of magnitude than any sensible estimate based on particle physics. In addition, the matter density ρ_M and and the vacuum energy ρ_Λ evolve at different rates, with $\rho_M/\rho_\Lambda \simeq R^{-3}$ and it would seem quite unlikely that they would differ today by a factor of order unity. Interesting attempts have been made during the past few years to justify this apparent fine tuning by incorporating evolving scalar fields (quintessence [17]) or probabilistic arguments based on the anthropic principle [18]).

In addition to the prediction for accelerating universe, the presence of a non-zero cosmological constant also affects the form of gravitational interactions. The generalized spherically symmetric vacuum solution of eq. (1.2) may be written as

$$ds^2 = A(r)c^2dt^2 - dr^2/A(r) - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1.4)$$

where $A(r) = 1 - 2GM/c^2r - \Lambda r^2/3$. This metric is known as the Schwarzschild-de-Sitter metric [19] and describes a space that is not asymptotically flat but has an asymptotic curvature induced by the vacuum energy corresponding to Λ. In the weak field limit we may use the Schwarzschild-de-Sitter metric to find the corresponding Newtonian potential ϕ

$$g_{00} = A(r) = 1 + 2\phi/c^2 \quad (1.5)$$

which leads to

$$\phi = \frac{GM}{r} + \frac{\Lambda c^2 r^2}{6} \quad (1.6)$$

This generalized Newtonian potential leads to a gravitational interaction acceleration

$$g = -\frac{GM}{r^2} + \frac{\Lambda c^2}{3} r \quad (1.7)$$

This generalized force includes a repulsive term

$$g_r = \frac{\Lambda c^2}{3} r \quad (1.8)$$

which is expected to dominate at distances larger than

$$r_c = (\frac{3GM}{\Lambda c^2})^{\frac{1}{3}} \simeq 10^2 (\frac{\bar{M}_1}{\bar{\Lambda}_{52}})^{\frac{1}{3}} pc \simeq 2 \times 10^7 (\frac{\bar{M}_1}{\bar{\Lambda}_{52}})^{\frac{1}{3}} AU \quad (1.9)$$

where $\bar{M}_1$ is the mass within a sphere of radius r_c in units of solar masses $M_\odot = 2 \times 10^{30} kg$ and $\bar{\Lambda}_{52}$ is the cosmological constant in units of $10^{-52} m^{-2}$.

The question we address in this paper is the following: 'What are the effects of the additional repulsive force g_r on the various astrophysical scales?' This issue has been addressed in the literature for particular scales. For example it was shown [13] that the effects of this term in the solar system could only become measurable (by modifying the perihelia precession) if the cosmological constant were fourteen orders of magnitude larger than the value implied by the SnIa observations.

A recent study [11] has also addressed this issue for galactic scales. In that study an attempt was made to explain the flat rotation curves of galaxies without the existence of dark matter. It was found that a cosmological constant $\Lambda \simeq 10^{-48} m^{-2}$ can account for the flat rotation curves of M33 and other galaxies and is consistent with a theoretical value obtained from the Extended Large Number Hypothesis. This value of Λ however is four orders of magnitude larger than that of eq. (1.1) indicated by supernova and other cosmological observations.

In the next section it will be shown that the vacuum energy required to close the universe (eq. (1.1)) has negligible effects on the dynamics of galactic scales (few tens of kpc). The dynamically derived mass to light ratios of galaxies obtained from velocity measurements on galactic scales are modified by less than 0.1% due to the vacuum energy term of eq. (1.1). This is not true however on cluster scales or larger. Even on the scales of the Local Group of galaxies (about 1Mpc) the gravitational effects of the vacuum energy are significant. We show that the dynamically obtained masses of M31 and the Milky Way must be increased by about 35% to compensate the repulsion of the vacuum energy of eq. (1.8) and produce the observed relative velocity of the members of the Local Group. The effects of vacuum energy are even more important on larger scales (rich cluster and supercluster).

II. SCALE DEPENDENCE OF ANTIGRAVITY

In order to obtain a feeling of the relative importance of antigravity vs gravity on the various astrophysical scales it is convenient to consider the ratio of the corresponding two terms in eq. (1.7). This ratio q may be written as

$$q = \frac{\Lambda c^2 r^3}{3GM} \simeq 0.5 \times 10^{-5} \frac{\bar{\Lambda}_{52} \bar{r}_1^3}{\bar{M}_1} \quad (2.1)$$

where $\bar{r}_1$ is the distance measured in units of pc. For the solar system ($\bar{r}_1 \simeq 10^{-5}$, $\bar{M}_1 = 1$) we have $q_{ss} \simeq 10^{-20}$ which justifies the fact that interplanetary measures can not give any useful bound on the cosmological constant.

For a galactic system ($\bar{r}_1 \simeq 10^4$, $\bar{M}_1 = 10^{10}$) we have $q_g \simeq 5 \times 10^{-4}$ which indicates that up to galactic scales the dynamical effects of the antigravity induced by Λ are negligible. On a cluster however ($\bar{r}_1 \simeq 10^7$, $\bar{M}_1 = 10^{14}$) we obtain $q_c \simeq O(1)$ and the gravitational effects of the vacuum energy become significant. This will be demonstrated in a more quantitative way in what follows.

The precessions of the perihelia of the planets provide one of the most sensitive Solar System tests for the cosmological constant. The additional precession due to the cosmological constant can be shown [13] to be

$$\Delta\phi_\Lambda = 6\pi q \; rad/orbit \quad (2.2)$$

where q is given by eq. (2.1). For Mercury we have $\bar{r}_1 \simeq 10^{-6}$ which leads to $q_{mc} \simeq 10^{-23}$ and $\Delta\phi_\Lambda \simeq$

$10^{-22} rad/orbit$. The uncertainty in the observed precession of the perihelion of Mercury is $0.1''$ per century or $\Delta\phi_{unc} \simeq 10^{-9} rad/orbit$ which is 13 orders of magnitude larger than the one required for the detection of a cosmologicaly interesting value for the cosmological constant. The precession per century[1] scales like $\bar{r}_1^{3/2}$ and therefore the predicted additional precession per century for distant planets ($\bar{r}_1(Pluto) \simeq 10^2 \bar{r}_1(Mercury)$) due to the cosmological constant increases by up to 3 orders of magnitude. It remains however approximatelly 10 orders of magnitude smaller than the precession required to give a cosmologically interesting detection of the cosmological constant even with the best quality of presently available observations. It is therefore clear that since the relative importance of the gravitational contribution is inversely proportional to the mean matter density on the scale considered, a cosmological constant could only have detectable gravitational effects on scales much larger than the scale of the solar system.

On galactic scales, the rotation velocities of spiral galaxies as measured in the 21cm line of neutral hydrogen comprise a good set of data for identifying the role of the vacuum energy on galactic scales. This is because these velocity fields usually extend well beyond the optical image of the galaxy on scales where the effects of Λ are maximized and because gas on very nearly circular orbits is a precise probe of the radial force law. For a stable circular orbit with velocity v_c at a distance r from the center of a galaxy with mass M we obtain using eq. (1.7)

$$v_c^2 = \frac{GM}{r} - \frac{\Lambda c^2 r^2}{3} \tag{2.3}$$

We now define the rescaled dimensionless quantities $\bar{v}_{100}$, $\bar{r}_{10}$ and $\bar{M}_{10}$ as follows:

$$\begin{aligned} v_c &= 100\ \bar{v}_{100}\ km/sec \\ r &= 10\ \bar{r}_{10}\ kpc \\ M &= 10^{10} \bar{M}_{10}\ M_\odot \end{aligned} \tag{2.4}$$

Eq. (2.3) may now be written in a rescaled form as

$$\bar{v}_{100}^2 = \frac{1}{2}\frac{\bar{M}_{10}}{\bar{r}_{10}} - 3 \times 10^{-5} \bar{\Lambda}_{52}\ \bar{r}_{10}^2 \tag{2.5}$$

In order to calculate the effects of the cosmological constant on the dynamically obtained masses of galaxies (including their halos) it is convenient to calculate the ratio

$$p \equiv \frac{M(\bar{\Lambda}_{52} = 1) - M(\bar{\Lambda}_{52} = 0)}{M(\bar{\Lambda}_{52} = 0)} = \frac{3 \times 10^{-5} \bar{r}_{10}^2}{\bar{v}_{100}^2} \tag{2.6}$$

In Table 1 we show a calculation of the mass ratio p for 22 galaxies of different sizes and rotation velocities [10]. The corresponding plot of $p(r)$ is shown in Fig. 1.

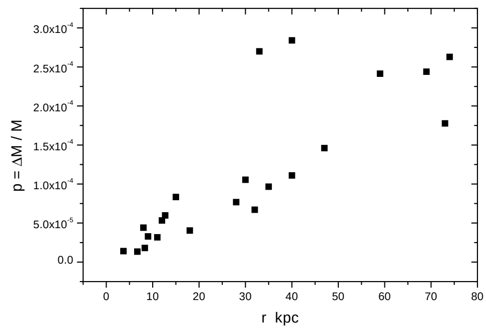

FIG. 1. Relative increase p of dynamically calculated mass of galaxies due to repusive effects vacuum energy vs galaxy radious r measured in kpc.

It is clear that even for large galaxies where the role of the repulsive force induced by vacuum energy is maximized, the increase of the mass needed to compensate vacuum energy antigravity is negligible. In order for these effects to be significant the cosmological constant would have to be larger that the value required for flatness by a factor of at least 10^3.

TABLE I. Relative increase p of dynamically calculated mass of galaxies due to repulsive effects of vacuum energy.

Galaxy	r_{HI} kpc	v_{rot} km/sec	p
UGC2885	73	300	17.8×10^{-5}
NGC5533	74	250	26.3×10^{-5}
NGC6674	69	242	24.3×10^{-5}
NGC5907	32	214	6.7×10^{-5}
NGC2998	47	213	14.6×10^{-5}
NGC801	59	208	24.1×10^{-5}
NGC5371	40	208	11.1×10^{-5}
NGC5033	35	195	9.7×10^{-5}
NGC3521	28	175	7.7×10^{-5}
NGC2683	18	155	4.0×10^{-5}
NGC6946	30	160	10.5×10^{-5}
UGC128	40	130	28.4×10^{-5}
NGC1003	33	110	27.0×10^{-5}
NGC247	11	107	3.1×10^{-5}
M33	8.3	107	1.8×10^{-5}
NGC7793	6.7	100	1.3×10^{-5}
NGC300	12.7	90	6.0×10^{-5}
NGC5585	12	90	5.3×10^{-5}
NGC2915	15	90	8.3×10^{-5}
NGC55	9	86	3.2×10^{-5}
IC2574	8	66	4.4×10^{-5}
DDO168	3.7	54	1.4×10^{-5}

[1] The angular velocity is smaller for distant planets and therefore the precession per century does not scale like the $\bar{r}_1^3$ as the precession per orbit does

Such value would be inconsistent with several cosmological observations even though it is consistent [11] with theoretical expectations based on the Extended Large Number Hypothesis. Therefore, even though the effects of vacuum energy on galactic dynamics are much more important compared to the corresponding effects on solar system dynamics it is clear that we must consider systems on even larger scales where the mean density is smaller in order to obtain any nontrivial effects on the dynamics.

The Local Group of galaxies is a particularly useful system for studying mass dynamics on large scales because it is close enough to be measured and modeled in detail yet it is large enough (and poor enough) to probe the effects of vacuum energy on the dynamics. The dominant members of the group are the Milky Way and the Andromeda Nebula M31. Their separation is

$$r_0 \equiv r(t=t_0) \simeq 800 kpc \tag{2.7}$$

and the rate of change of their separation is

$$\frac{dr}{dt}(t=t_0) \simeq -123 kms^{-1} \tag{2.8}$$

A widely used assumption is that the motion of approach of M31 and Milky Way is due to the mutual gravitational attraction of the masses of the two galaxies. Adopting the simplest model of the Local Group as an isolated two body system, the Milky Way and M31 have negligible relative angular momentum and their initial rate of change of separation is zero in comoving coordinates. The equation of motion for the separation $r(t)$ of the centers of the two galaxies in the presence of a nonzero cosmological constant is:

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2} + \frac{\Lambda c^2}{3} r \tag{2.9}$$

where M is the sum of the masses of the two galaxies. A similar equation (with $\Lambda = 0$) was used in Ref [20] to obtain an approximation of the mass to light ratio of the galaxies of the Local Group. Numerical studies [21] have shown that this approximation is reasonable and leads to a relatively small overestimation (about 25%) of the galactic masses. This correction is due to the effects of the other dwarf members of the Local Group that are neglected in the isolated two body approximation. Here we are not interested in the precise evaluation of the masses of the galaxies but on the effects of the cosmological constant on the evaluation of these masses. Therefore we will use the 'isolated two body approximation' of the Local Group (eq. (2.9)) and focus on the dependence of the calculated value of the mass M as a function of Λ in the range of cosmologicaly interesting values of Λ. Our goal is to find the total mass M of the Local Group galaxies, using eq. (2.9) supplied with the following conditions:

$$r(t=t_0) = 800\ kpc \tag{2.10}$$

$$\frac{dr}{dt}(t=t_0) = -123\ km/sec \tag{2.11}$$

$$\frac{dr}{dt}(t=0) = 0 \tag{2.12}$$

where $t_0 = 15Gyr$. Upon integrating and rescaling eq. (2.9) we obtain

$$\left(\frac{d\bar{r}_{100}}{d\bar{t}_{15}}\right)^2 = \bar{M}\left(\frac{1}{\bar{r}_{100}} - \frac{1}{8}\right) + \bar{\Lambda}(\bar{r}_{100}^2 - 64) + 420 \equiv f(\bar{r}_{100}) \tag{2.13}$$

where we have used condition (2.11) and the rescaled quantities defined by

$$r = 100\ \bar{r}_{100}\ kpc \tag{2.14}$$

$$t = 1.5 \times 10^{10}\ \bar{t}_{15}\ yrs \tag{2.15}$$

$$M = 4 \times 10^8\ \bar{M}\ M_\odot \tag{2.16}$$

$$\Lambda = 1.3\ \bar{\Lambda}_{52} \tag{2.17}$$

Using now conditions (2.10) and (2.12) we obtain the equation that can be solved to evaluate the galactic masses for various Λ

$$1 = \bar{t}_{15}(t=t_0) = -\int_{\bar{r}_{100}(t=0)}^{\bar{r}_{100}(t=t_0)} \frac{dr}{\sqrt{f(\bar{r}_{100})}} \tag{2.18}$$

The lower limit of the integral (2.18) is obtained by solving condition (2.12) for r (using eq. (2.13) while the upper limit is given by eq. (2.10) in its rescaled form. This equation can be solved numerically for M to calculate the galactic total mass M for various values of the cosmological constant Λ. The resulting dependence of M on Λ is shown in Fig. 2 (continuous line).

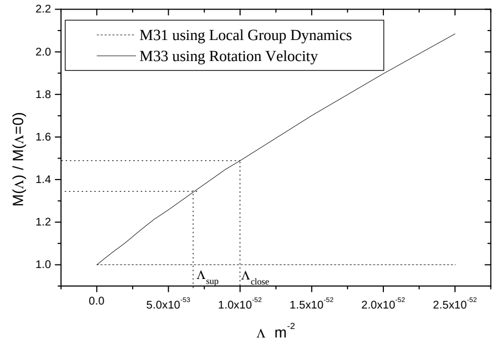

FIG. 2. The relative galactic total mass $\frac{M(\Lambda)}{M(\Lambda=0)}$ calculated for various values of the cosmological constant Λ using Local Group (continous line) and galactic scale (M33, dashed line) velocity data. The dotted lines correspond to the Λ value implied by the SnIa data and to Λ_{cl} for which the vacuum energy alone closes the universe.

Clearly, for a value of Λ consistent with the recent SnIa observations ($\Lambda \simeq 0.7 \times 10^{-52} m^{-52}$) the calculated galactic masses using Local Group dynamics are 35% larger than the corresponding masses calculated with $\Lambda = 0$. On Fig. 2 we also plot (dashed line) the dependence of the $\frac{M(\Lambda)}{M(\Lambda=0)}$ on Λ calculated using galactic dynamics (rotation velocity) of the galaxy M33. Clearly a galactic system in contrast to the Local Group is too small and dense to be a sensitive detector of the cosmological constant.

III. CONCLUSION

We conclude that the Local Group of galaxies is a system that is large enough and with low enough matter density to be a sensitive probe of the gravitational effects of a cosmological constant with value consistent with cosmological expectations and recent SnIa obsrevations. Smaller and denser systems do not have this property. On the other hand the gravitational effects of Λ should be even more pronounced on larger cosmological systems.

The specific cluster dynamic effect which is discussed here offers an independent determination of the cosmological constant contribution to a galactic 'dark matter' halo. This is consistent with the fact that galactic masses of galaxies like our Milky Way can be deduced from cluster dynamics such as our Local Group as well as by using non-dynamical methods (e.g. gravitational lensing).

It is legitimate to wonder whether the distinctive action of the cosmological constant over small versus larger scales persists if a different form of vacuum energy such as 'quintessence' [22] is dominant. This type of effective 'scalar matter' possesses an equation of state $p = w\rho$ ($-1 < w < 0$) and is much softer than the one associated with the cosmological constant ($w = -1$). As such it interpolates between the latter a normal dark matter component ($w \geq 0$). Moreover we should expect that it causes a much smaller repulsion effect smoothly passing over to a purely dark matter for increasing w. As a consequence our analysis for such a 'reduced' type of antigravity would most probably imply a successively smaller discrepancy for the relative total galactic mass contribution of quintessence as derived from the Local Group versus the galactic velocity data for increasing w ($-1 < w < 0$). A recent analysis of the dynamical effects of quintessence on flat galactic rotation curves [23] in fact corroborates to this point of view.

The gravitational effects discussed here can only be used as an independent detection method of the cosmological constant, if the galactic masses of systems like the Local Group are measured independently using non-dynamical methods (eg gravitational lensing). In that case Λ could be obtained using plots like the one shown in Fig. 2. This type of investigation is currently in progress.

IV. ACKNOWLEDGEMENTS

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