The role of an anomalous abelian symmetry in the fermion mass hierarchy and the problem of proton decay

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- G.K. Leontaris and J. Rizos, *Nucl. Phys.* **B567**,(2000)32, hep-ph/9909206
- J. Ellis, G.K. Leontaris and J. Rizos, hep-ph/0002263

▶ The Standard Model and some questions

The Standard Model and its extension the MSSM is a rather successful theory, but it leaves unanswered some questions like

1. How to explain the fermion mass hierarchy ?

2. We need extra symmetries (R-parity) to eliminate unwanted couplings.

3. The MSSM does not include gravity.

Is it possible to solve these problems by extending the MSSM gauge symmetry?

$$SU(3)\times SU(2)\times U(1)_Y \rightarrow SU(3)\times SU(2)\times U(1)_Y\times U(1)'$$

Is this extension arbitrary ?

What about GUTs?

$$SU(5)\times U(1) \rightarrow SU(5)\times U(1)\times U(1)'$$

▶ The fermion mass hierarchy-Textures

The fermion masses arise from the couplings

$$\mathcal{W} = \lambda^u_{ij}\, Q_i U^c_j H_2 + \lambda^d_{ij}\, Q_i D^c_j H_1 + \lambda^e_{ij}\, E_i E^c_j H_1$$

$$\begin{array}{rrl} \text{up quarks} & M_u \sim & \lambda^u_{ij}\langle H_2\rangle \\ \text{down quarks} & M_d \sim & \lambda^d_{ij}\langle H_1\rangle \\ \text{leptons} & M_e \sim & \lambda^e_{ij}\langle H_1\rangle \end{array}$$

According to the analysis of P. Ramond, R.G. Roberts and G.G. Ross, (1993) it is enough to have

Texture	M_U	M_D
T_1	$\begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix}$
T_2	$\begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 2\lambda^3 & 1 \end{pmatrix}$
T_3	$\begin{pmatrix} 0 & 0 & \sqrt{2}\lambda^4 \\ 0 & \lambda^4 & 0 \\ \sqrt{2}\lambda^4 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix}$
T_4	$\begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \sqrt{3}\lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
T_5	$\begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \sqrt{2}\lambda^4 & \frac{\lambda^2}{\sqrt{2}} \\ \lambda^4 & \frac{\lambda^2}{\sqrt{2}} & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

in order to reproduce the correct quark mass matrices. (λ is a small parameter e.g. $\lambda \sim 0.7$)

► Lepton and Baryon number violating interactions

The gauge symmetries of the MSSM allow the following terms in the Lagrangian which lead to violation of lepton and baryon number

$$\mathcal{W}_R = \lambda_{ijk}\, L_i L_j E^c_k + \lambda'_{ijk}\, L_i Q_j D^c_k + \lambda''_{ijk}\, U^c_i D^c_j D^c_k + \mu_i L_i H_2$$

The experimental constrains are very strict

$$\lambda, \lambda', \lambda'' < 10^{-21} - 10^{-6}$$

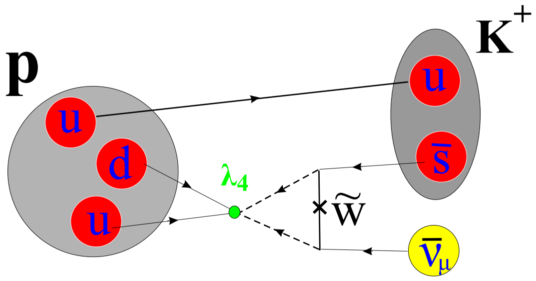
Usually we eliminate these terms by introducing R-parity

$$R = (-1)^{3B+L+2S}$$

But R-parity does not eliminate the terms

$$\frac{\lambda_4^{ijkl}}{M} Q_i Q_j Q_k L_l \ , \ \frac{\lambda_5^{ijkl}}{M} U^c_i U^c_j D^c_k E^c_l$$

which appear as non-renormalizable interactions in the presence of gravity (which becomes strong at the scale M). These terms lead to proton decay



Typical diagram of proton decay through dim-5 operators

To render these terms harmless we need for $M \sim 10^{18} GeV$

$$\lambda_4 < 10^{-7}$$

▶ Other interactions allowed by R-parity

R-parity allows also a left-handed neutrino (Majorana) mass term

$$\frac{\lambda_8}{M}(L_i H_2)(L_j H_2)$$

and the μ-term

$$\mu H_1 H_2$$

If $\mu > 100 GeV$ the Higgs mechanism does not work. On the other hand this term is necessary for SUSY breaking.

In the context of effective theories of gravity incorporating the MSSM the μ term appears either as tree level or NR interaction and for string theories with siglet field vevs $\langle\phi\rangle \sim 10^{-1} M$

$$\frac{\langle\phi\rangle^n}{M^{n-1}} H_2 H_1$$

we need to eliminate these terms up to 16$^{\text{th}}$ order G.K. Leontaris J. Rizos and J. Ellis, *Phys. Lett.* **B464** (1999)62

▶ Anomalies

Anomalies appear when promoting a classical theory to a quantum field theory.

Typical anomaly diagram

anomalies of local symmetries $= 0 \longrightarrow$ contraints on field theory

In the MSSM we find to kind of anomalies

$$D_{\alpha\beta\gamma} \sim \text{tr}\left(\{T_\alpha, T_\beta\}T_\gamma\right)$$
$$\delta_a \sim \text{tr}\left(\text{T}_\alpha\right)$$

Where T_α is a generator of the gauge symmetry $SU(3)\times SU(2)\times U(1)_Y$

Solving these anomaly constrains one can determine almost uniquely the MSSM particle hyperchage assignment. (see Weinberg book)

▶ Anomalies in String Theory

In string theory there is more freedom

$$\mathrm{tr}\left(U(1)'^{3}\right) \neq 0$$
$$\mathrm{tr}\left(U(1)'\right) \neq 0$$

since this anomaly is canceled by the Dine-Seiberg-Witten mechanism which also gives vevs to some charged fields

$$\langle\phi\rangle \sim \frac{1}{10} M_{\mathrm{string}} \sim 10^{17} GeV$$

These anomalies are related with MSSM quantities

$$\frac{A_3}{A_2} = \frac{k_3}{k_2}$$
$$\frac{A_2}{A_1} = \frac{k_2}{k_1}$$

$$\begin{aligned} A_3 &= SU(3) - SU(3) - U(1)' \\ A_2 &= SU(2) - SU(2) - U(1)' \\ A_1 &= U(1)_Y - U(1)_Y - U(1)' \end{aligned}$$

k_3, k_2, k_1 the Kac-Moody levels of $SU(3), SU(2), U(1)$

The standard unification scenario demands

$$\begin{aligned} \sin^2\theta_W &= \frac{k_1 + k_2}{k_2} = \frac{3}{8} \\ k_3 &= k_2 \end{aligned}$$

leading to the contraints

$$\frac{A_3}{A_1} = \frac{A_2}{A_1} = \frac{3}{5}$$

▶ Introducing an additional $U(1)$ symmetry

We now consider the MSSM with an additional anomalous $U(1)'$

$$SU(3) \times SU(2) \times U(1) \times U(1)'$$

and two extra MSSM siglet fields $\phi, \bar{\phi}$ with charges $+1, -1$ under $U(1)'$ These fields develop vevs through the DSW mechanism $\langle\phi\rangle, \langle\bar{\phi}\rangle$

$$\varepsilon = \frac{\langle\phi\rangle}{M}\ , \bar{\varepsilon} = \frac{\langle\bar{\phi}\rangle}{M} \quad M \sim M_{\text{Planck}}$$

$Q_i(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	q_i
$U_i^c(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	u_i
$D_i^c(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	d_i
$L_i(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	ℓ_i
$E_i^c(\mathbf{1}, \mathbf{1}, +\mathbf{1})$	e_i
$H_1(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	h_1
$\bar{H}_2(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	h_2

$Q_3 U_3 H_2 \quad q_3 + u_3 + h_2 = 0$
$Q_2 U_2 H_2 \left(\frac{<\phi>}{M}\right)^2 \sim \frac{1}{10} Q_2 U_2 H_2 \quad q_2 + u_2 + h_2 = -2$

▶ Contraints

- Symmetric mass matrices

$$\begin{aligned} q_i + u_j &= q_j + u_i \\ q_i + d_j &= q_j + d_i \\ \ell_i + e_j &= \ell_j + e_i . \end{aligned}$$

- The heaviest family acquires mass at tree-level (large $\tan\beta$)

$$\begin{aligned} q_3 + u_3 + h_2 &= 0 \\ q_3 + d_3 + h_1 &= 0 \\ \ell_3 + e_3 + h_1 &= 0 \end{aligned}$$

- Anomalies

$A_3 = SU(3) - SU(3) - U(1)'$, $A_2 = SU(2) - SU(2) - U(1)'$
$A_1 = U(1)_Y - U(1)_Y - U(1)'$, $A_0 = U(1)_Y - U(1) - U(1)'$

$$A_3 = \sum q_i + \frac{1}{2}\sum (u_i + d_i)$$
$$A_2 = \frac{3}{2}\sum q_i + \frac{1}{2}\sum \ell_i + \frac{1}{2}(h_1 + h_2)$$
$$A_1 = \frac{1}{6}\sum q_i + \frac{1}{3}\sum d_i + \frac{4}{3}\sum u_i + \frac{1}{2}\sum \ell_i + \sum e_i + \frac{1}{2}(h_1 + h_2)$$
$$A_0 = \sum q_i^2 + \sum d_i^2 - 2\sum u_i^2 - \sum \ell_i^2 + \sum e_i^2 + (h_2^2 - h_1^2) = 0$$

- Unification, Weinberg angle

$$\frac{A_3}{A_1} = \frac{A_2}{A_1} = \frac{3}{5} .$$

► The general solution of the conditions

$$C^{QU^cH_2} =$$
$$C^{QD^cH_1} = \begin{pmatrix} 2(q_1-q_3) & (q_1-q_3)+(q_2-q_3) & q_1-q_3 \\ (q_1-q_3) & 2(q_2-q_3) & q_2-q_3 \\ (q_1-q_3) & (q_2-q_3) & 0 \end{pmatrix}$$

$$q_1 - q_3 = \frac{n}{2},\; q_2 - q_3 = \frac{m}{2},\quad \begin{array}{l} m,n = \pm 1, \pm 2, \ldots \\ m+n \neq 0 \end{array}$$

$$C^{LE^cH_1} = \begin{pmatrix} 2(\ell_1-\ell_3) & (\ell_1-\ell_3)+(\ell_2-\ell_3) & \ell_1-\ell_3 \\ (\ell_1-\ell_3) & 2(\ell_2-\ell_3) & \ell_2-\ell_3 \\ (\ell_1-\ell_3) & (\ell_2-\ell_3) & 0 \end{pmatrix}$$

to obtain an acceptable lepton matrix

$$h_+ = h_1 + h_2 = \text{integer}$$
$$k = 2\ell_2 + 6q_3 - m + \frac{7}{3}h_+ = \text{integer}$$

Field	Family		
	1	2	3
Q	$\frac{n}{2}+q_3$	$\frac{m}{2}+q_3$	q_3
U^c	$\frac{n}{2}-q_3-h_2$	$\frac{m}{2}-q_3-h_2$	$-q_3-h_2$
D^c	$\frac{n}{2}-q_3-h_++h_2$	$\frac{m}{2}-q_3-h_+-h_2$	$-h_++h_2-q_3$
L	$\frac{n-k}{2}-3q_3-7\frac{h_+}{6}$	$\frac{m+k}{2}-3q_3-7\frac{h_+}{6}$	$-3q_3-7\frac{h_+}{6}$
E^c	$\frac{n-k}{2}+3q_3-\frac{h_+}{3}+h_2$	$\frac{m+k}{2}+3q_3+\frac{h_+}{6}+h_2$	$\frac{h_+}{6}+h_2+q_3$
Higgs			
H_1	h_+-h_2	H_2	h_2

$U(1)'$ charges of the general solution ($h_+ \neq m+n$) which depends on four integer m, n, h_+, k and one real parameter q_3. h_2 is determined by solving (??).

$$h_2 = \frac{-10\,h_+^2 + h_+(m+n-54\,q_3) + 24q_3(m+n)}{12(h_+-m-n)} \qquad (1)$$

▶ Charges of R-parity violating couplings

Coupling	Charge
$L_1Q_1D_1^c$	$\frac{3n-k}{2} - 5\,q_3$
$L_1Q_1D_2^c$, $L_1Q_2D_1^c$	$n + \frac{m-k}{2} - 5\,q_3$
$L_1Q_1D_3^c$, $L_1Q_3D_1^c$	$n - \frac{k}{2} - 5\,q_3$
$L_1Q_2D_2^c$	$m + \frac{n-k}{2} - 5\,q_3$
$L_1Q_2D_3^c$, $L_1Q_3D_2^c$	$\frac{m+n-k}{2} - 5\,q_3$
$L_1Q_3D_3^c$	$\frac{n-k}{2} - 5\,q_3$
$L_2Q_1D_1^c$	$n + \frac{k+m}{2} - 5\,q_3$
$L_2Q_1D_2^c$,$L_2Q_2D_1^c$	$m + \frac{k+n}{2} - 5\,q_3$
$L_2Q_1D_3^c$, $L_2Q_3D_1^c$	$\frac{k+m+n}{2} - 5\,q_3$
$L_2Q_2D_2^c$	$\frac{k+3\,m}{2} - 5\,q_3$
$L_2Q_2D_3^c$, $L_2Q_3D_2^c$	$m + \frac{k}{2} - 5\,q_3$
$L_2Q_3D_3^c$	$\frac{k+m}{2} - 5\,q_3$
$L_3Q_1D_1^c$	$n - 5\,q_3$
$L_3Q_1D_2^c$, $L_3Q_2D_1^c$	$\frac{m+n}{2} - 5\,q_3$
$L_3Q_1D_3^c$, $L_3Q_3D_1^c$	$\frac{n}{2} - 5\,q_3$
$L_3Q_2D_2^c$	$m - 5\,q_3$
$L_3Q_2D_3^c$, $L_3Q_3D_2^c$	$\frac{m}{2} - 5\,q_3$
$L_3Q_3D_3^c$	$-5\,q_3$

Coupling	Charge
$L_1L_2E_1^c$	$n + \frac{m-k}{2} - 5\,q_3$
$L_1L_2E_2^c$	$m + \frac{n+k}{2} - 5\,q_3$
$L_1L_2E_3^c$	$\frac{m+n}{2} - 5\,q_3$
$L_1L_3E_1^c$	$n - k - 5\,q_3$
$L_1L_3E_2^c$	$\frac{m+n}{2} - 5\,q_3$
$L_1L_3E_3^c$	$\frac{n-k}{2} - 5\,q_3$
$L_2L_3E_1^c$	$\frac{m+n}{2} - 5\,q_3$
$L_2L_3E_2^c$	$k + m - 5\,q_3$
$L_2L_3E_3^c$	$\frac{k+m}{2} - 5\,q_3$

Coupling	Charge
$U_1^cD_1^cD_2^c$	$n + \frac{m}{2} - 5\,q_3$
$U_1^cD_1^cD_3^c$	$n - 5\,q_3$
$U_1^cD_2^cD_3^c$	$\frac{m+n}{2} - 5\,q_3$
$U_2^cD_1^cD_2^c$	$m + \frac{n}{2} - 5\,q_3$
$U_2^cD_1^cD_3^c$	$\frac{m+n}{2} - 5\,q_3$
$U_2^cD_2^cD_3^c$	$m - 5\,q_3$
$U_3^cD_1^cD_2^c$	$\frac{m+n}{2} - 5\,q_3$
$U_3^cD_1^cD_3^c$	$\frac{n}{2} - 5\,q_3$
$U_3^cD_2^cD_3^c$	$\frac{m}{2} - 5\,q_3$

Coupling	Charge
L_1H_2	$\frac{n-k}{2} - 5q_3$
L_2H_2	$\frac{m+k}{-}5q_3$
L_3H_2	$-5q_3$

Charge assignments of R-parity violating couplings in the case $h_+ = 0$.

▶ Dimension 5 operators

coupling		charge	m, n even	m even, n odd	n, m odd
$Q_1Q_1Q_2L_1$	$D_1^cU_1^cU_2^cE_1$	$\frac{3n+m-k}{2}$		$\surd$	
$Q_1Q_1Q_3L_1$	$D_1^cU_1^cU_3^cE_1$	$\frac{3n-k}{2}$			
$Q_1Q_2Q_3L_1$	$D_1^cU_2^cU_3^cE_1$ $D_2^cU_1^cU_3^cE_1$ $D_3^cU_1^cU_2^cE_1$	$n+\frac{m-k}{2}$		$\surd$	$\surd$
$Q_2Q_1Q_2L_1$	$D_2^cU_1^cU_2^cE_1$	$m+n-\frac{k}{2}$			
$Q_2Q_2Q_3L_1$	$D_2^cU_2^cU_3^cE_1$	$m+\frac{n-k}{2}$			
$Q_3Q_1Q_3L_1$	$D_3^cU_1^cU_3^cE_1$	$n-\frac{k}{2}$			
$Q_3Q_2Q_3L_1$	$D_3^cU_2^cU_3^cE_1$	$\frac{m+n-k}{2}$			
$Q_1Q_1Q_2L_2$	$D_1^cU_1^cU_2^cE_2$	$n+m+\frac{k}{2}$			
$Q_1Q_1Q_3L_2$	$D_1^cU_1^cU_3^cE_2$	$n+\frac{k+m}{2}$		$\surd$	$\surd$
$Q_1Q_2Q_3L_2$	$D_1^cU_2^cU_3^cE_2$ $D_2^cU_1^cU_3^cE_2$ $D_3^cU_1^cU_2^cE_2$	$m+\frac{k+n}{2}$		$\surd$	$\surd$
$Q_2Q_1Q_2L_2$	$D_2^cU_1^cU_2^cE_2$	$\frac{n+3m+k}{2}$		$\surd$	
$Q_2Q_2Q_3L_2$	$D_2^cU_2^cU_3^cE_2$	$\frac{3m+k}{2}$		$\surd$	$\surd$
$Q_3Q_1Q_3L_2$	$D_3^cU_1^cU_3^cE_2$	$\frac{m+k+n}{2}$		$\surd$	
$Q_3Q_2Q_3L_2$	$D_3^cU_2^cU_3^cE_2$	$m+\frac{k}{2}$			
$Q_1Q_1Q_2L_3$	$D_1^cU_1^cU_2^cE_3$	$n+\frac{m}{2}$	$\surd$		
$Q_1Q_1Q_3L_3$	$D_1^cU_1^cU_3^cE_3$	n	$\surd$	$\surd$	$\surd$
$Q_1Q_2Q_3L_3$	$D_1^cU_2^cU_3^cE_3$ $D_2^cU_1^cU_3^cE_3$ $D_3^cU_1^cU_2^cE_3$	$\frac{m+n}{2}$	$\surd$		$\surd$
$Q_2Q_1Q_2L_3$	$D_2^cU_1^cU_2^cE_3$	$m+\frac{n}{2}$	$\surd$	$\surd$	
$Q_2Q_2Q_3L_3$	$D_2^cU_2^cU_3^cE_3$	m	$\surd$	$\surd$	$\surd$
$Q_3Q_1Q_3L_3$	$D_3^cU_1^cU_3^cE_3$	$\frac{n}{2}$	$\surd$	$\surd$	
$Q_3Q_2Q_3L_3$	$D_3^cU_2^cU_3^cE_3$	$\frac{m}{2}$	$\surd$		

Charges of dimension five operators in the case $h_+ = 0$. We indicate with the $\surd$ the surviving operators in the case k =even. The charges of the case $h_+ \neq 0$ is obtained by adding $\frac{5}{3}h_+$ or $\frac{7}{6}h_+$

▶ Some typical solutions : Solution A

Solution A			
Field	Family		
	1	2	3
Q	4	2	0
U^c	4	2	0
D^c	4	2	0
L	4	2	0
E^c	4	2	0
Higgs			
H_1	0	H_2	0
Singlets			
ϕ	1	$\bar{\phi}$	-1

$$M_u = \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^6 & \bar{\epsilon}^4 \\ \bar{\epsilon}^6 & \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^4 & \bar{\epsilon}^2 & 1 \end{pmatrix}$$

$$M_d = \begin{pmatrix} \bar{\lambda}^8 & \bar{\lambda}^6 & \bar{\lambda}^4 \\ \bar{\lambda}^6 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \bar{\lambda}^4 & \bar{\lambda}^2 & 1 \end{pmatrix}$$

✓ The dim-5 operators are acceptable

$$\begin{aligned} \lambda_4^{3233} &\sim \bar{\lambda}^2 \\ \lambda_4^{3133}, \lambda_4^{3223}, \lambda_4^{3232} &\sim \bar{\lambda}^4 \\ \lambda_4^{3231}, \lambda_4^{3222}, \lambda_4^{3132}, \lambda_4^{1233} &\sim \bar{\lambda}^6 \\ \lambda_4^{2231}, \lambda_4^{3131}, \lambda_4^{1232}, \lambda_4^{1133}, \lambda_4^{2123} &\sim \bar{\lambda}^8 \\ \lambda_4^{1231}, \lambda_4^{1132}, \lambda_4^{2122}, \lambda_4^{1123} &\sim \bar{\lambda}^{10} \\ \lambda_4^{1131}, \lambda_4^{2121}, \lambda_4^{1122} &\sim \bar{\lambda}^{12} \\ \lambda_4^{1121} &\sim \bar{\lambda}^{14} \end{aligned}$$

✓ Neutrino mass terms

$$L_i L_j H_2 H_2 \sim \frac{m_W^2}{M_U} \begin{pmatrix} \bar{\lambda}^8 & \bar{\lambda}^6 & \bar{\lambda}^4 \\ \bar{\lambda}^6 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \bar{\lambda}^4 & \bar{\lambda}^2 & 1 \end{pmatrix}$$

✗ The lepton and baryon number violating interactions are present, thus we need R-parity.

▶ Some typical solutions : Solution B

Solution B			
field	family		
	1	2	3
Q	6	4	2
D^c	−2	−4	−6
U^c	6	4	2
L	−2	−4	−6
E^c	6	4	2
Higgs			
H_1	4	H_2	−4
Singlets			
ϕ	1	$\bar{\phi}$	−1

$$M_u = \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^6 & \bar{\epsilon}^4 \\ \bar{\epsilon}^6 & \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^4 & \bar{\epsilon}^2 & 1 \end{pmatrix}$$

$$M_d = \begin{pmatrix} \bar{\lambda}^8 & \bar{\lambda}^6 & \bar{\lambda}^4 \\ \bar{\lambda}^6 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \bar{\lambda}^4 & \bar{\lambda}^2 & 1 \end{pmatrix}$$

✓ Dim-5 operators are similar with solution A.

∗There are no Majorana neutrino mass terms

× The lepton and baryon number violating interactions are present, thus we need R-parity.

× No suppression of the μ term

▶ Some typical solutions : Solution C

Solution C			
field	family		
	1	2	3
Q	$\frac{9}{2}$	$\frac{5}{2}$	$\frac{1}{2}$
D^c	$\frac{5}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$
U^c	$\frac{9}{2}$	$\frac{5}{2}$	$\frac{1}{2}$
L	$\frac{5}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$
E^c	$\frac{9}{2}$	$\frac{5}{2}$	$\frac{1}{2}$
Higgs			
H_1	1	H_2	-1
Singlets			
ϕ	1	$\bar{\phi}$	-1

$$M_u = \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^6 & \bar{\epsilon}^4 \\ \bar{\epsilon}^6 & \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^4 & \bar{\epsilon}^2 & 1 \end{pmatrix}$$

$$M_d = \begin{pmatrix} \bar{\lambda}^8 & \bar{\lambda}^6 & \bar{\lambda}^4 \\ \bar{\lambda}^6 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \bar{\lambda}^4 & \bar{\lambda}^2 & 1 \end{pmatrix}$$

✓ Dim-5 operators are similar with solution A.

✓ All R-parity violating couplings are eliminated

✗ The μ term is not suppressed

✓ There are neutrino mass terms

$$L_i L_j H_2 H_2 \sim \frac{M_w^2}{M_U} \begin{pmatrix} \bar{\lambda}^3 & \bar{\lambda}^1 & \lambda^1 \\ \bar{\lambda}^1 & \lambda^1 & \lambda^3 \\ \lambda^1 & \lambda^3 & \lambda^5 \end{pmatrix}$$

▶ Some typical solutions : Solution D

Solution D			
field	family		
	1	2	3
Q	$\frac{7}{3}$	$\frac{5}{6}$	$-\frac{2}{3}$
D^c	5	$\frac{7}{2}$	2
U^c	$\frac{7}{3}$	$\frac{5}{6}$	$-\frac{2}{3}$
L	5	$\frac{7}{2}$	2
E^c	$\frac{7}{3}$	$\frac{5}{6}$	$-\frac{2}{3}$
Higgs			
H_1	$-\frac{4}{3}$	H_2	$\frac{4}{3}$
Singlets			
ϕ	1	$\bar{\phi}$	-1

$$M_u = \begin{pmatrix} \bar{\epsilon}^6 & 0 & \bar{\epsilon}^3 \\ 0 & \bar{\epsilon}^3 & 0 \\ \bar{\epsilon}^3 & 0 & 1 \end{pmatrix}$$

$$M_d = \begin{pmatrix} \bar{\lambda}^6 & 0 & \bar{\lambda}^3 \\ 0 & \bar{\lambda}^3 & 0 \\ \bar{\lambda}^3 & 0 & 1 \end{pmatrix}$$

✓ Dim-5 operators are eliminated

✓ All R-parity violating couplings are eliminated

× The μ term is not suppressed

∗ No neutrino Majorana mass terms

▶ Some typical solutions : Solution E

Solution E			
field	family		
	1	2	3
Q	$-\frac{1}{2}$	$-\frac{5}{2}$	$-\frac{9}{2}$
D^c	$-\frac{17}{6}$	$-\frac{29}{6}$	$-\frac{41}{6}$
U^c	$\frac{23}{6}$	$\frac{11}{6}$	$-\frac{1}{6}$
L	$-\frac{55}{6}$	$-\frac{19}{6}$	$-\frac{31}{6}$
E^c	$-\frac{61}{6}$	$-\frac{25}{6}$	$-\frac{37}{6}$
Higgs			
H_1	$\frac{34}{3}$	H_2	$\frac{14}{3}$
Singlets			
ϕ	1	$\bar{\phi}$	-1

$$M_u = \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^6 & \bar{\epsilon}^4 \\ \bar{\epsilon}^6 & \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^4 & \bar{\epsilon}^2 & 1 \end{pmatrix}$$

$$M_d = \begin{pmatrix} \bar{\lambda}^8 & \bar{\lambda}^6 & \bar{\lambda}^4 \\ \bar{\lambda}^6 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \bar{\lambda}^4 & \bar{\lambda}^2 & 1 \end{pmatrix}$$

$$M_\ell = \begin{pmatrix} \lambda^8 & \lambda^2 & \lambda^4 \\ \lambda^2 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \lambda^4 & \bar{\lambda}^2 & 1 \end{pmatrix}$$

✓ All dim-5 operators are eliminated

✓ All R-parity violating couplings are eliminated

× The μ term is suppressed

∗ No neutrino Majorana mass terms

▶ Application to GUTs, the $SU(5)\times U(1)$ example

$SU(5)\times U(1)$	$SU(3)\times SU(2)\times U(1)$
$F_i\left(10,\frac{1}{2}\right)$	$Q(3,2,\frac{1}{6})+d^c(\bar{3},1,\frac{1}{3})+\nu^c(1,1,0)$
$\bar{f}_i\left(\bar{5},-\frac{3}{2}\right)$	$u^c(\bar{3},1,-\frac{2}{3})+\ell(1,2,-\frac{1}{2})$
$e_i^c\left(1,\frac{5}{2}\right)$	$e^c(1,1,1)$
$H\left(10,\frac{1}{2}\right)$	$Q_H(3,2,\frac{1}{6})+d^c_H(\bar{3},1,\frac{1}{3})+\nu^c_H(1,1,0)$
$\bar{H}\left(\overline{10},-\frac{1}{2}\right)$	$\bar{Q}_H(3,2,-\frac{1}{6})+\bar{d}^c_H(\bar{3},1,-\frac{1}{3})+\bar{\nu}^c_H(1,1,0)$
$h\,(5,-1)$	$D(\bar{3},1,-\frac{1}{3})+h_d(2,1,-\frac{1}{2})$
$\bar{h}\left(\bar{5},1\right)$	$\bar{D}(\bar{3},1,\frac{1}{3})+h_u(2,1,\frac{1}{2})$

Decomposition of the $SU(5)\times U(1)'$ matter and Higgs into representations of the Standard Model gauge group

Fermion mass terms

$$\begin{aligned} FFh &\to qd^ch_d+d^c\nu^cD+qqD \\ F\bar{f}\bar{h} &\to qu^ch_u+\ell\nu^ch_u+q\bar{D}\ell+d^cu^c\bar{D} \\ \bar{f}h\ell^c &\to \ell e^ch_d \end{aligned}$$

R-parity violating terms

$$\frac{H}{M}FF\bar{f} \to \frac{\langle\nu^c_H\rangle}{M}(\ell qd^c, u^cd^cd^c)$$

$$\frac{H}{M}\bar{f}\bar{f}\ell^c \to \frac{\langle\nu^c_H\rangle}{M}\ell\ell e^c$$

Dim-5 operators

$$\frac{\lambda_4^{ijkl}}{M_U}\,F_iF_jF_k\bar{f}_l,\quad \frac{\lambda_5^{ijkl}}{M_U}\,F_i\bar{f}_j\bar{f}_k\ell^c_l$$

Triplet–double splitting

$$HHh+\bar{H}\bar{H}\bar{h}$$

▶ Flipped $SU(5)$ specific examples

field	generation		
	1	2	3
F	$\frac{n}{2}+\alpha_3$	$\frac{m}{2}+\alpha_3$	α_3
$\bar{f}$	$\frac{n}{2}-\bar{\epsilon}-\alpha_3$	$\frac{m}{2}-\bar{\epsilon}-\alpha_3$	$-\bar{\epsilon}-\alpha_3$
ℓ^c	$\frac{n}{2}+\bar{\epsilon}+3\alpha_3$	$\frac{m}{2}+\bar{\epsilon}+3\alpha_3$	$\bar{\epsilon}+3\alpha_3$
Higgs			
H	δ	$\bar{H}$	$\bar{\delta}$
h	$-2\alpha_3$	$\bar{h}$	$\bar{\epsilon}$

The $U(1)'$ charge assignments for the general solution in the symmetric mass case in flipped $SU(5)$. The variables $\delta, \bar{\delta}$ are not free parameters, but can be expressed in terms of $\alpha_3, \bar{\epsilon}$. Taking into account triplet doublet–splitting we can express the above solution in terms of 4 integer parameters.

field	generation		
	1	2	3
F	2	0	-2
$\bar{f}$	2	0	-2
ℓ^c	2	0	-2
Higgs			
H	$-\frac{3}{2}$	$\bar{H}$	$\frac{3}{2}$
h	4	$\bar{h}$	4

field	generation		
	1	2	3
F	$-\frac{3}{2}$	-3	$-\frac{9}{2}$
$\bar{f}$	$\frac{3}{2}$	0	$-\frac{3}{2}$
ℓ^c	$-\frac{9}{2}$	-6	$-\frac{15}{2}$
Higgs			
H	0	$\bar{H}$	-6
h	9	$\bar{h}$	6

$$\frac{m_Q}{\langle h\rangle}, \frac{m_d}{\langle \bar{h}\rangle}, \frac{m_\ell}{\langle \bar{h}\rangle} \propto \begin{pmatrix} \rho^8 & \rho^6 & \rho^4 \\ \rho^6 & \rho^4 & \rho^2 \\ \rho^4 & \rho^2 & 1 \end{pmatrix}, \begin{pmatrix} \rho^6 & 0 & \rho^3 \\ 0 & \rho^3 & 0 \\ \rho^3 & 0 & 1 \end{pmatrix}, \begin{pmatrix} \rho^{16} & \rho^6 & \rho^8 \\ \rho^6 & \rho^4 & \rho^2 \\ \rho^4 & \rho^2 & 1 \end{pmatrix}$$

▶ References

1. C.D. Froggatt and H.B. Nielsen, Nucl. Phys. **B147** (1979) 277.

2. L. Ibanez and G.G. Ross, Phys. Lett. **B332** (1994) 100.

3. M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. **B420** (1994) 468.

4. Y. Nir, Phys. Lett. **B354** (1995) 107.

5. Y. Grossman and Y. Nir, Nucl. Phys. **B448** (1995) 30

6. P. Binetruy and P. Ramond, Phys. Lett. **B350** (1995)

7. H. Dreiner, G.K. Leontaris, S. Lola, G.G. Ross and C. Scheich, Nucl. Phys. **B436** (1995) 461

8. P. Binetruy, S. Lavignac and P. Ramond, Nucl. Phys. **B477** (1996) 353.

9. C.D. Froggatt , hep-ph/9811461

10. M.E. Gomez, G.K. Leontaris, S. Lola , J.D. Vergados, Phys.Rev.D59:116009,1999 .

11. J. Ellis, M.E. Gomez, G.K. Leontaris, S. Lola (CERN), D.V. Nanopoulos, hep-ph/9911459

12. G. Altarelli and F. Feruglio,Phys. Lett. **B451** (1999) 388

13. J. M. Mira, E. Nardi , D. A. Restrepo , hep-ph/9911212

14. L. E. Ibanez, F. Quevedo, JHEP 9910:001,1999

▶ Conclusions

$SU(3) \times SU(2) \times U(1)_Y \rightarrow SU(3) \times SU(2) \times U(1)_Y \times U(1)'$

The introduction of an additional anomalous $U(1)'$ symmetry can explain

✓ The problem of fermion mass hierarchy

✓ The absence of lepton and baryon number violating couplings

✓ Provide neutrino masses

✓ The suppression of the Higgs mixing term ($\mu - term$), can be explained only with exotic charge assignments

$SU(5) \times U(1) \rightarrow SU(5) \times U(1) \times U(1)'$

✓ Similar results hold for the flipped $SU(5)$ but the number of possible solutions and mass matrix textures is now reduced due to GUT relations so models become more predictive.

It is interesting to note that if we lower the string scale (see Brane scenarios) the above mechanism can be used to prevent baryon and lepton number violation.