The Directional Rate and the Modulation Effect for Direct Supersymmetric Matter Detection.

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Abstract

The detection of the theoretically expected dark matter is central to particle physics and cosmology. Current fashionable supersymmetric models provide a natural dark matter candidate which is the lightest supersymmetric particle (LSP). Such models combined with fairly well understood physics like the quark substructure of the nucleon and the nuclear structure (form factor and/or spin response function), permit the evaluation of the event rate for LSP-nucleus elastic scattering. The thus obtained event rates are, however, very low or even undetectable. So it is imperative to exploit the modulation effect, i.e. the dependence of the event rate on the earth's annual motion. Also it is useful to consider the directional rate, i.e its dependence on the direction of the recoiling nucleus. In this paper we study such a modulation effect both in non directional and directional experiments. We calculate both the differential and the total rates using both isothermal, symmetric as well as only axially asymmetric, and non isothermal, due to caustic rings, velocity distributions. We consider We find that in the symmetric case the modulation amplitude is small. The same is true for the case of caustic rings. The inclusion of asymmetry, with a realistic enhanced velocity dispersion in the galactocentric direction, yields an enhanced modulation effect, especially in directional experiments.

1 Introduction

In recent years the consideration of exotic dark matter has become necessary in order to close the Universe [1], [2]. Furthermore in in order to understand the large scale structure of the universe it has become necessary to consider matter made up of particles which were non-relativistic at the time of freeze out. This is the cold dark matter component (CDM). The COBE data [3] suggest that CDM is at least 60% [4]. On the other hand during the last few years evidence has appeared from two different teams, the High-z Supernova Search Team [5] and the Supernova Cosmology Project [6] , [7] which suggests that the Universe may be dominated by the cosmological constant Λr. As a matter of fact recent data the situation can be adequately described by a barionic component $\Omega_B = 0.1$ along with the exotic components $\Omega_{CDM} = 0.3$ and $\Omega_\Lambda = 0.6$. In another analysis Turner [8] gives $\Omega_m = \Omega_{CDM} + \Omega_B = 0.4$. Since the non exotic component cannot exceed 40% of the CDM [2], [9], there is room for the exotic WIMP's (Weakly Interacting Massive Particles). In fact the DAMA experiment [10] has claimed the observation of one signal in direct detection of a WIMP, which with better statistics has subsequently been interpreted as a modulation signal [11].

The above developments are in line with particle physics considerations. Thus, in the currently favored supersymmetric (SUSY) extensions of the standard model, the most natural WIMP candidate is the LSP, i.e. the lightest supersymmetric particle. In the most favored scenarios the LSP can be simply described as a Majorana fermion, a linear combination of the neutral components of the gauginos and Higgsinos [2], [12]-[23].

Since this particle is expected to be very massive, $m_\chi \geq 30GeV$, and extremely non relativistic with average kinetic energy $T \leq 100KeV$, it can be directly detected [12]-[13] mainly via the recoiling of a nucleus (A,Z) in the elastic scattering process:

$$\chi + (A, Z) \to \chi + (A, Z)^* \tag{1}$$

(χ denotes the LSP). In order to compute the event rate one needs the following ingredients:

1) An effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described in Refs. [2], Bottino *et al.* [20] and [23].

2) A procedure in going from the quark to the nucleon level, i.e. a quark model for the nucleon. The results depend crucially on the content of the nucleon in quarks other than u and d. This is particularly true for the scalar couplings as well as the isoscalar axial coupling [14, 25].

3) Compute the relevant nuclear matrix elements [26]-[30] using as reliable as possible many body nuclear wave functions. By putting as accurate nuclear physics input as possible, one will be able to constrain the SUSY parameters as much as possible. The situation is a bit simpler in the case of the scalar coupling, in which case one only needs the nuclear form factor.

Since the obtained rates are very low, one would like to be able to exploit the modulation of the event rates due to the earth's revolution around the sun. To this end one adopts a folding procedure assuming some distribution [2, 31, 32] of velocities for the LSP. One also would like to know the directional rates, by observing the nucleus in a certain direction, which correlate with the motion of the sun around the center of the galaxy.

The purpose of our present review is to focus on this last point along the lines suggested by our recent work [17, 18]. For the reader's convenience, however, we will give a description in sects 2, and 4 of the basic SUSY ingredients needed to calculate LSP-nucleus scattering cross section. We will not, however, elaborate on how one gets the needed parameters from supersymmetry. The calculation of these parameters has become pretty standard. One starts with representative input in the restricted SUSY parameter space as described in the literature, e.g. Bottino *et al.* [20], Kane *et al.*, Castano *et al.* and Arnowitt *et al.* [21]. Our own SUSY input parameters will appear elsewhere [19]

After this we will specialize our study in the case of the nucleus ^{127}I, which is one of the most popular targets [10],[33] [34]- [36]. To this end we will consider velocity distributions both isothermal, symmetric Maxwell-Boltzmann [2] as well as only axially asymmetric, like that of Drukier [32] and non isothermal as well. In the non isothermal case we will consider the relatively simple case in which dark matter accumulated as late in-fall of such matter into our galaxy, i.e. the Sikivie caustic rings [31].

Since the expected rates are extremely low or even undetectable with present techniques, one would like to exploit the characteristic signatures provided by the reaction. Such are: a) The modulation effect, i.e the dependence of the event rate on the velocity of the Earth and b) The directional event rate, which depends on the velocity of the sun around the galaxy as well as the the velocity of the Earth. The latter effect, recognized sometime ago [37] has recently begun to appear feasible by the planned UKDMC experiment [38]. We will study both of these effects in the present work.

In all calculations we will, of course, include an appropriate nuclear form factor and take into account the influence on the rates of the detector energy cut off. We will present our results a function of the LSP mass, m_χ, in a way which can be easily understood by the experimentalists.

2 The Nature of the LSP

Before proceeding with the construction of the effective Lagrangian we will briefly discuss the nature of the lightest supersymmetric particle (LSP) focusing on those ingredients which are of interest to dark matter.

In currently favorable supergravity models the LSP is a linear combination [2, 12] of the neutral four fermions $\tilde{B}, \tilde{W}_3, \tilde{H}_1$ and $\tilde{H}_2$ which are the supersymmetric partners of the gauge bosons B_μ and W_μ^3 and the Higgs scalars H_1 and H_2. Admixtures of s-neutrinos are expected to be negligible.

In the above basis the mass-matrix takes the form [2, 23]

$$\begin{pmatrix} M_1 & 0 & -m_z c_\beta s_w & m_z s_\beta s_w \\ 0 & M_2 & m_z c_\beta c_w & -m_z s_\beta c_w \\ -m_z c_\beta s_w & m_z c_\beta c_w & 0 & -\mu \\ m_z s_\beta s_w & -m_z c_\beta c_w & -\mu & 0 \end{pmatrix} \tag{2}$$

In the above expressions $c_W = cos\theta_W$, $s_W = sin\theta_W$, $c_\beta = cos\beta$, $s_\beta = sin\beta$, where $tan\beta = \langle \upsilon_2 \rangle / \langle \upsilon_1 \rangle$ is the ratio of the vacuum expectation values of the Higgs scalars H_2 and H_1. μ is a dimensionful coupling constant which is not specified by the theory (not even its sign). The parameters $tan\beta, M_1, M_2, \mu$ are determined by the procedure of Kane it et al and Castano *et al* in Ref. [21] using universal masses of the GUT scale.

By diagonalizing the above matrix we obtain a set of eigenvalues m_j and the diagonalizing matrix C_{ij} as follows

$$\begin{pmatrix} \tilde{B}_R \\ \tilde{W}_{3R} \\ \tilde{H}_{1R} \\ \tilde{H}_{2R} \end{pmatrix} = (C_{ij}^R) \begin{pmatrix} \chi_{1R} \\ \chi_{2R} \\ \chi_{3R} \\ \chi_{4R} \end{pmatrix} \qquad \begin{pmatrix} \tilde{B}_L \\ \tilde{W}_{2L} \\ \tilde{H}_{1L} \\ \tilde{H}_{2L} \end{pmatrix} = (C_{ij}) \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \\ \chi_{3L} \\ \chi_{4L} \end{pmatrix} \tag{3}$$

with $C_{ij}^R = C_{ij}^* e^{i\lambda_j}$ The phases are $\lambda_i = 0, \pi$ depending on the sign of the eigemass.

Another possibility to express the above results in photino-zino basis $\tilde{\gamma}, \tilde{Z}$ via

$$\begin{aligned} \tilde{W}_3 &= sin\theta_W \tilde{\gamma} - cos\theta_W \tilde{Z} \\ \tilde{B}_0 &= cos\theta_W \tilde{\gamma} + sin\theta_W \tilde{Z} \end{aligned} \tag{4}$$

In the absence of supersymmetry breaking ($M_1 = M_2 = M$ and $\mu = 0$) the photino is one of the eigenstates with mass M. One of the remaining eigenstates has a

Table 1: The essential parameters describing the LSP and Higgs. For the definitions see the text.

Solution	#1	#2	#3	#4	#5	#6	#7	#8	#9
$m_x\,(GeV)$	126	27	102	80	124	58	34	35	50
m_h	116.0	110.2	113.2	124.0	121.0	105.0	103.0	92.0	111.0
m_H	345.6	327.0	326.6	595.0	567.0	501.0	184.0	228.0	234.0
m_A	345.0	305.0	324.0	594.0	563.0	497.0	179.0	207.0	230.0
tan2α	0.245	6.265	0.525	0.410	0.929	0.935	0.843	1.549	0.612
tanβ	10.0	1.5	5.0	5.4	2.7	2.7	5.2	2.6	5.3

zero eigenvalue and is a linear combination of $\tilde{H}_1$ and $\tilde{H}_2$ with mixing $sin\beta$. In the presence of SUSY breaking terms the $\tilde{B}, \tilde{W}_3$ basis is superior since the lowest eigenstate χ_1 or LSP is primarily $\tilde{B}$. From our point of view the most important parameters are the mass m_x of LSP and the mixings $C_{j1}, j = 1,2,3,4$ which yield the χ_1 content of the initial basis states. These parameters which are relevant here are shown in Table 1. We are now in a position to find the interaction of χ_1 with matter. We distinguish three possibilities involving Z-exchange, s-quark exchange and Higgs exchange.

3 The Feynman Diagrams Entering the Direct Detection of LSP.

The diagrams involve Z-exchange, s-quark exchange and Higgs exchange.

3.1 The Z-exchange contribution.

This can arise from the interaction of Higgsinos with Z which can be read from Eq. C86 of Ref. [23]

$$L = \frac{g}{cos\theta_W}\frac{1}{4}[\tilde{H}_{1R}\gamma_\mu\tilde{H}_{1R} - \tilde{H}_{1L}\gamma_\mu\tilde{H}_{1L} - (\tilde{H}_{2R}\gamma_\mu\tilde{H}_{2R} - \tilde{H}_{2L}\gamma_\mu\tilde{H}_{2L})]Z^\mu \tag{5}$$

Using Eq. (3) and the fact that for Majorana particles $\bar{\chi}\gamma_\mu\chi = 0$, we obtain

$$L = \frac{g}{cos\theta_W}\frac{1}{4}(|C_{31}|^2 - |C_{41}|^2)\bar{\chi}_1\gamma_\mu\gamma_5\chi_1 Z^\mu \tag{6}$$

which leads to the effective 4-fermion interaction

$$L_{eff} = \frac{g}{cos\theta_W}\frac{1}{4}2(|C_{31}|^2 - |C_{41}|^2)(-\frac{g}{2cos\theta_W}\frac{1}{q^2 - m_Z^2}\bar{\chi}_1\gamma^\mu\gamma_5\chi_1)J_\mu^Z \tag{7}$$

where the extra factor of 2 comes from the Majorana nature of χ_1. The neutral hadronic current J^Z_λ is given by

$$J^Z_\lambda = -\bar{q}\gamma_\lambda\{\frac{1}{3}sin^2\theta_W - \Big[\frac{1}{2}(1-\gamma_5) - sin^2\theta_W\Big]\tau_3\}q \tag{8}$$

at the nucleon level it can be written as

$$\tilde{J}^Z_\lambda = -\bar{N}\gamma_\lambda\{\, sin^2\theta_W - g_V(\frac{1}{2} - sin^2\theta_W)\tau_3 + \frac{1}{2}g_A\gamma_5\tau_3\}N \tag{9}$$

Thus we can write

$$L_{eff} = -\frac{G_F}{\sqrt{2}}(\bar{\chi}_1\gamma^\lambda\gamma^5\chi_1)J_\lambda(Z) \tag{10}$$

where

$$J_\lambda(Z) = \bar{N}\gamma_\lambda[f^0_V(Z) + f^1_V(Z)\tau_3 + f^0_A(Z)\gamma_5 + f^1_A(Z)\gamma_5\tau_3]N \tag{11}$$

and

$$f^0_V(Z) = 2(|C_{31}|^2 - |C_{41}|^2)\frac{m^2_Z}{m^2_Z - q^2}sin^2\theta_W \tag{12}$$

$$f^1_V(Z) = -2(|C_{31}|^2 - |C_{41}|^2)\frac{m^2_Z}{m^2_Z - q^2}g_V(\frac{1}{2} - sin^2\theta_W) \tag{13}$$

$$f^0_A(Z) = 0 \tag{14}$$

$$f^1_A(Z) = 2(|C_{31}|^2 - |C_{41}|^2)\frac{m^2_Z}{m^2_Z - q^2}\,\frac{1}{2}g_A \tag{15}$$

with $g_V = 1.0, g_A = 1.24$. We can easily see that

$$f^1_V(Z)/f^0_V(Z) = -g_V(\frac{1}{2sin^2\theta_W} - 1) \simeq -1.15 \tag{16}$$

Note that the suppression of this Z-exchange interaction compared to the ordinary neutral current interactions arises from the smallness of the mixings C_{31} and C_{41}, a consequence of the fact that the Higgsinos are normally quite a bit heavier than the gauginos. Furthermore, the two Higgsinos tend to cancel each other.

3.2 The s-quark Mediated Interaction

The other interesting possibility arises from the other two components of χ_1, namely $\tilde{B}$ and $\tilde{W}_3$. Their corresponding couplings to s-quarks can be read from the appendix C4 of Ref. [23] They are

$$\begin{aligned} L_{eff} &= -g\sqrt{2}\{\bar{q}_L[T_3\tilde{W}_{3R} - tan\theta_W(T_3 - Q)\tilde{B}_R]\tilde{q}_L \\ &- tan\theta_W\bar{q}_R Q\tilde{B}_L\tilde{q}_R\} + HC \end{aligned} \tag{17}$$

where $\tilde{q}$ are the scalar quarks (SUSY partners of quarks). A summation over all quark flavors is understood. Using Eq. (3) we can write the above equation in the χ_i basis. Of interest to us here is the part

$$\begin{aligned} L_{eff} &= g\sqrt{2}\{(tan\theta_W(T_3 - Q)C^R_{11} - T_3C^R_{21})\bar{q}_L\chi_{1R}\tilde{q}_L \\ &+ tan\theta_W C_{11}Q\bar{q}_R\chi_{1L}\tilde{q}_R\} \end{aligned} \tag{18}$$

The above interaction is almost diagonal in the quark flavor. There exists, however, mixing between the s-quarks $\tilde{q}_L$ and $\tilde{q}_R$ (of the same flavor) i.e.

$$\tilde{q}_L = cos\theta_{\tilde{q}}\tilde{q}_1 + sin\theta_{\tilde{q}}\tilde{q}_2 \tag{19}$$

$$\tilde{q}_R = -sin\theta_{\tilde{q}}\tilde{q}_1 + cos\theta_{\tilde{q}}\tilde{q}_2 \tag{20}$$

with

$$tan2\theta_{\tilde{u}} = \frac{m_u(A + \mu cot\beta)}{m^2_{u_L} - m^2_{\tilde{u}_R} + m^2_z cos2\beta/2} \tag{21}$$

$$tan2\theta_{\tilde{d}} = \frac{m_d(A + \mu tan\beta)}{m^2_{d_R} - m^2_{\tilde{d}_R} + m^2_Z cos2\beta/2} \tag{22}$$

Thus Eq. (18) becomes

$$\begin{aligned} L_{eff} &= g\sqrt{2}\,\{[B_L cos\theta_{\tilde{q}}\ \bar{q}_L\chi_{1R} - B_R sin\theta_{\tilde{q}}\bar{q}_R\chi_{1L}]\tilde{q}_1 \\ &+ \ [B_L sin\theta_{\tilde{q}}\bar{q}_L\chi_{1R} + B_R cos\theta_{\tilde{q}}\bar{q}_R\chi_{1L}]\ \tilde{q}_2\} \end{aligned}$$

with

$$B_L(q) = -\tfrac{1}{6}C^R_{11}tan\theta_\omega - \tfrac{1}{2}C^R_{21}, \ \ q = u \ \ (charge \ \ 2/3)$$

$$B_L(q) = -\tfrac{1}{6}C^R_{11}tan\theta_\omega + \tfrac{1}{2}C^R_{21}, \ \ \ q = d \ \ (charge \ \ -1/3)$$

$$B_R(q) = \tfrac{2}{3}tan\theta_\omega C_{11}, \ \ \ q = u \ \ (charge \ \ 2/3)$$

$$B_R(q) = -\tfrac{1}{3}tan\theta_\omega C_{11}, \ \ \ q = d \ \ (charge \ \ -1/3)$$

The effective four fermion interaction takes the form

$$\begin{aligned} L_{eff} = (g\sqrt{2})^2\{&(B_L cos\theta_{\tilde{q}}\bar{q}_L\chi_{1R} - B_R sin\theta_{\tilde{q}}\bar{q}_R\chi_{1L}) \\ &\frac{1}{q^2 - m_{\tilde{q}^2_1}}(B_L cos\theta_q\bar{\chi}_{1R}q_L - B_R sin\theta_{\tilde{q}}\bar{\chi}_{1L}q_R) \\ &+(B_L sin\theta_q q_L\chi_{1R} + cos\theta_{\tilde{q}}\bar{q}_R\chi_{1L})\frac{1}{q^2 - m_{\tilde{q}^2_2}} \\ &(B_L sin\theta_q\bar{\chi}_{1R}q_L + B_R cos\theta_{\tilde{q}}\bar{\chi}_{1L}q_R)\} \end{aligned} \tag{23}$$

The above effective interaction can be written as

$$L_{eff} = L^{LL+RR}_{eff} + L^{LR}_{eff} \tag{24}$$

The first term involves quarks of the same chirality and is not much effected by the mixing (provided that it is small). The second term involves quarks of opposite chirality and is proportional to the s-quark mixing.

i) The part L^{LL+RR}_{eff}

Employing a Fierz transformation L_{eff}^{LL+RR} can be cast in the more convenient form

$$\begin{aligned} L_{eff}^{LL+RR} = & \quad (g\sqrt{2})^2 2(-\frac{1}{2})\{|B_L|^2 \\ & \quad (\frac{cos^2\theta_{\tilde{q}}}{q^2 - m_{\tilde{q}_1^2}} + \frac{sin^2\theta_{\tilde{q}}}{q^2 - m_{\tilde{q}_2^2}})\bar{q}_L\gamma_\lambda q_L \chi_{1R}\gamma^\lambda \chi_{1R} \\ & \quad +|B_R|^2(\frac{sin^2\theta_{\tilde{q}}}{q^2 - m_{\tilde{q}_1^2}} + \frac{cos^2\theta_{\tilde{q}}}{q^2 - m_{\tilde{q}_2^2}})\bar{q}_R\gamma_\lambda q_R \chi_{1L}\gamma^\lambda \chi_{1L}\} \end{aligned} \tag{25}$$

The factor of 2 comes from the Majorana nature of LSP and the (-1/2) comes from the Fierz transformation. Equation (25) can be written more compactly as

$$\begin{aligned} L_{eff} &= -\frac{G_F}{\sqrt{2}} 2\{\bar{q}\gamma_\lambda(\beta_{0R} + \beta_{3R}\tau_3)(1+\gamma_5)q \\ &- \bar{q}\gamma_\lambda(\beta_{0L} + \beta_{3L}\tau_3)(1-\gamma_5)q\}(\bar{\chi}_1\gamma^\lambda\gamma^5\chi_1\} \end{aligned} \tag{26}$$

with

$$\begin{aligned} \beta_{0R} &= \Big(\frac{4}{9}\chi^2_{\tilde{u}_R} + \frac{1}{9}\chi^2_{\tilde{d}_R}\Big)|C_{11}tan\theta_W|^2 \\ \beta_{3R} &= \Big(\frac{4}{9}\chi^2_{\tilde{u}_R} - \frac{1}{9}\chi^2_{\tilde{d}_R}\Big)|C_{11}tan\theta_W|^2 \\ \beta_{0L} &= |\frac{1}{6}C^R_{11}tan\theta_W + \frac{1}{2}C^R_{21}|^2\chi^2_{\tilde{u}_L} + |\frac{1}{6}C^R_{11}tan\theta_W - \frac{1}{2}C^R_{21}|^2\chi^2_{\tilde{d}_L} \\ \beta_{3L} &= |\frac{1}{6}C^R_{11}tan\theta_W + \frac{1}{2}C^R_{21}|^2\chi^2_{\tilde{u}_L} - |\frac{1}{6}C^R_{11}tan\theta_W - \frac{1}{2}C^R_{21}|^2\chi^2_{\tilde{d}_L} \end{aligned} \tag{27}$$

with

$$\begin{aligned} \chi^2_{qL} &= c^2_{\tilde{q}}\frac{m^2_W}{m_{\tilde{q}_1^2} - q^2} + s^2_{\tilde{q}}\frac{m^2_W}{m_{\tilde{q}_2^2} - q^2} \\ \chi^2_{qR} &= s^2_{\tilde{q}}\frac{m^2_W}{m_{\tilde{q}_1^2} - q^2} + c^2_{\tilde{q}}\frac{m^2_W}{m_{\tilde{q}_2^2} - q^2} \\ c_{\tilde{q}} &= cos\theta_{\tilde{q}}, \;\; s_{\tilde{q}} = sin\theta_{\tilde{q}} \end{aligned} \tag{28}$$

The above parameters are functions of the four-momentum transfer which in our case is negligible. Proceeding as in Sec. 2.2.1 we can obtain the effective Lagrangian at the nucleon level as

$$L_{eff}^{LL+RR} = -\frac{G_F}{\sqrt{2}}(\bar{\chi}_1\gamma^\lambda\gamma^5\chi_1)J_\lambda(\tilde{q}) \tag{29}$$

$$J_\lambda(\tilde{q}) = \bar{N}\gamma_\lambda\{f^0_V(\tilde{q}) + f^1_V(\tilde{q})\tau_3 + f^0_A(\tilde{q})\gamma_5 + f^1_A(\tilde{q})\gamma_5\tau_3)N \tag{30}$$

with

$$\begin{aligned} f^0_V = 6(\beta_{0R} - \beta_{0L}), &\qquad f^1_V = 2(\beta_{3R} - \beta_{3L}) \\ f^0_A = 2g^0_a g_V(\beta_{0R} + \beta_{0L}), &\qquad f^1_A = 2g_A(\beta_{3R} + \beta_{3L}) \end{aligned} \tag{31}$$

with $g_v = 1.0$ and $g_A = 1.25$. The quantity g^0_A depends on the quark model for the nucleon. It can be anywhere between 0.12 and 1.00.

We should note that this interaction is more suppressed than the ordinary weak interaction by the fact that the masses of the s-quarks are usually larger than that of the gauge boson Z^0. In the limit in which the LSP is a pure bino ($C_{11}=1, C_{21}=0$) we obtain

$$
\begin{aligned}
\beta_{0R} &= tan^2\theta_W\Big(\frac{4}{9}\chi^2_{u_R}+\frac{1}{9}\chi^2_{\tilde{d}_R}\Big)\\
\beta_{3R} &= tan^2\theta_W\Big(\frac{4}{9}\chi^2_{u_R}-\frac{1}{9}\chi^2_{\tilde{d}_R}\Big)\\
\beta_{0L} &= \frac{tan^2\theta_W}{36}(\chi^2_{\tilde{u}_L}+\chi^2_{\tilde{d}_L})\\
\beta_{3L} &= \frac{tan^2\theta_W}{36}(\chi^2_{\tilde{u}_L}-\chi^2_{\tilde{d}_L})
\end{aligned}
\tag{32}
$$

Assuming further that $\chi_{\tilde{u}_R}=\chi_{\tilde{d}_R}=\chi_{\tilde{u}_L}=\chi_{\tilde{d}_L}$ we obtain

$$
\begin{aligned}
f^1_V(\tilde{q})/f^0_V(\tilde{q}) &\simeq +\frac{2}{9}\\
f^1_A(\tilde{q})/f^0_A(\tilde{q}) &\simeq +\frac{6}{11}
\end{aligned}
\tag{33}
$$

If, on the other hand, the LSP were the photino ($C_{11}=cos\theta_W, C_{21}=sin\theta_W, C_{31}=C_{41}=0$) and the s-quarks were degenerate there would be no coherent contribution ($f^0_V=0$ if $\beta_{0L}=\beta_{0R}$).

ii) L^{LR}_{eff}

From Eq. (23) we obtain

$$
L^{LR}_{eff}=-(g\sqrt{2})^2 sin2\theta_{\tilde{q}}B_L(q)B_R(q)\frac{1}{2}[\frac{1}{q^2-m_{\tilde{q}^2_1}}-\frac{1}{q^2-m_{\tilde{q}^2_2}}]
$$
$$
(\bar{q}_L\chi_{1R}\bar{\chi}_{1L}q_R+\bar{q}_R\chi_{1L}\bar{\chi}_{1R}q_L)
$$

Employing a Fierz transformation we can cast it in the form

$$
\begin{aligned}
L_{eff} = -\frac{G_F}{\sqrt{2}}[&\beta_+(q)(\bar{q}q\bar{\chi}_1\chi_1+\bar{q}\gamma_5 q\bar{\chi}_1\gamma_5\chi_1-(\bar{q}\sigma_{\mu\nu}q)(\bar{\chi}_1\sigma^{\mu\nu}\chi_1))\\
&+\beta_-(\bar{q}\tau_3 q\bar{\chi}_1\chi_1+\bar{q}\tau_3\gamma_5 q\bar{\chi}_1\gamma_5\chi_1-\bar{q}\sigma_{\mu\nu}\tau_3 q\bar{\chi}_1\sigma^{\mu\nu}\chi_1)]
\end{aligned}
$$

where for the light quarks u and d

$$
\begin{aligned}
\beta_\pm &= \frac{1}{3}tan\theta_W C_{11}\{2sin2\theta_{\tilde{u}}[\frac{1}{6}C^R_{11}tan\theta_W+\frac{1}{2}C^R_{21}]\Delta_{\tilde{u}}\\
&\mp\ sin2\theta_{\tilde{d}}[\frac{1}{6}C^R_{11}tan\theta_W-\frac{1}{2}C^R_{21}]\Delta_{\tilde{d}}\}
\end{aligned}
$$

for quarks other than u and d we only have only the isoscalar contribution which is given by

$$
\begin{aligned}
\beta_+ &= \frac{2}{3}tan\theta_W C_{11}\{2sin2\theta_{\tilde{u}}[\frac{1}{6}CR_{11}tan\theta_W+\frac{1}{2}CR_{21}]\Delta_{\tilde{u}}\\
&+\ sin2\theta_{\tilde{d}}[\frac{1}{6}CR_{11}tan\theta_W-\frac{1}{2}C^R_{21}]\Delta_{\tilde{d}}\}
\end{aligned}
$$

Where in the last expression u indicates quarks with charge 2/3 and d quarks with charge -1/3. In all cases

$$\Delta_{\tilde{u}} = \frac{(m^2_{\tilde{u}_1} - m^2_{\tilde{u}_2})M^2_W}{(m^2_{\tilde{u}_1} - q^2)(m^2_{\tilde{u}_2} - q^2)}$$

and an analogous equation for $\Delta_{\tilde{d}}$.

The appearance of scalar terms in s-quark exchange has been first noticed by Griest. [16] It has also been noticed there that one should consider explicitly the effects of quarks other than u and d [14] in going from the quark to the nucleon level. We first notice that with the exception of t s-quark the $\tilde{q}_L - \tilde{q}_R$ mixing small. Thus

$$\begin{aligned} sin2\theta_{\tilde{u}}\Delta\tilde{u} &\simeq \frac{2m_u(A + \mu cot\beta)m^2_W}{(m^2_{\tilde{u}_L} - q^2)(m^2_{\tilde{u}_R} - q^2)} \\ sin2\theta_{\tilde{d}}\Delta\tilde{d} &\simeq \frac{2m_d(A + \mu tan\beta)m^2_W}{(m^2_{\tilde{d}_L} - q^2)(m^2_{\tilde{d}_R} - q^2)} \end{aligned}$$

In going to the nucleon level and ignoring the negligible pseudoscalar and tensor components we only need modify the above expressions for all quarks other than t by the substitution $m_q \rightarrow f_q m_N$. We will see in the next section that the quarks s,c and b tend to dominate. For the t s-quark the mixing is complete, which implies that the amplitude is independent of the top quark mass. Hence in the case of the top quark we do not get an extra enhancement in going from the quark to the nucleon level. In any case this way we get

$$L_{eff} = \frac{G_F}{\sqrt{2}}[f^0_s(\tilde{q})\bar{N}N + f^1_s(\tilde{q})\bar{N}\tau_3 N]\bar{\chi}_1\chi_1 \tag{34}$$

with

$$f^0_s(\tilde{q}) = f^0_q\beta_+ \quad and \quad f^1_s(\tilde{q}) = f^1_q\beta_- \tag{35}$$

(see sect. 3.3 for details). In the allowed SUSY parameter space considered in this work this contribution can be neglected in front of the Higgs exchange contribution. This happens because for quarks other than t the s-quark mixing is small. For the t-quark, as it has already been mentioned, we have large mixing, but we do not get the advantage of the mass enhancement.

3.3 The Intermediate Higgs Contribution

The coherent scattering can be mediated via the intermediate Higgs particles which survive as physical particles . The relevant interaction can arise out of the Higgs-Higgsino-gaugino interaction which takes the form

$$\begin{aligned} L_{H\chi\chi} &= \frac{g}{\sqrt{2}}\Big(\bar{\tilde{W}}^3_R\tilde{H}_{2L}H^{0*}_2 - \bar{\tilde{W}}^3_R\tilde{H}_{1L}H^{0*}_1 \\ &- tan\theta_w(\bar{\tilde{B}}_R\tilde{H}_{2L}H^{0*}_2 - \bar{\tilde{B}}_R\tilde{H}_{1L}H^{0*}_1)\Big) + H.C. \end{aligned} \tag{36}$$

Proceeding as above we can express $\tilde{W}$ an $\tilde{B}$ in terms of the appropriate eigenstates and retain the LSP to obtain

$$\begin{aligned} L &= \frac{g}{\sqrt{2}}\Big((C^R_{21} - tan\theta_w C^R_{11})C_{41}\bar{\chi}_{1R}\chi_{1L}H^{o*}_2 \\ &- (C^R_{21} - tan\theta_w C^R_{11})C_{31}\bar{\chi}_{1R}\chi_{1L}H^{o*}_1\Big) + H.C. \end{aligned} \tag{37}$$

We can now proceed further and express the fields H_1^{0*}, H_2^{0*} in terms of the physical fields h, H and A. The term which contains A will be neglected, since it yields only a pseudoscalar coupling which does not lead to coherence.

Thus we can write

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}}\bar{\chi}\chi\,\bar{N}[f_s^0(H) + f_s^1(H)\tau_3]N \tag{38}$$

where

$$f_s^0(H) = \frac{1}{2}(g_u + g_d) + g_s + g_c + g_b + g_t \tag{39}$$

$$f_s^1(H) = \frac{1}{2}(g_u - g_d) \tag{40}$$

with

$$g_{a_i} = [g_1(h)\frac{cos\alpha}{sin\beta} + g_2(H)\frac{sin\alpha}{sin\beta}]\frac{m_{a_i}}{m_N}, \quad a_i = u, c, t \tag{41}$$

$$g_{\kappa_i} = [-g_1(h)\frac{sin\alpha}{cos\beta} + g_2(H)\frac{cos\alpha}{cos\beta}]\frac{m_{\kappa_i}}{m_N}, \quad \kappa_i = d, s, b \tag{42}$$

$$g_1(h) = 4(C_{11}^R tan\theta_W - C_{21}^R)(C_{41}cos\alpha + C_{31}sin\alpha)\frac{m_N m_W}{m_h^2 - q^2} \tag{43}$$

$$g_2(H) = 4(C_{11}^R tan\theta_W - C_{21}^R)(C_{41}sin\alpha - C_{31}cos\alpha)\frac{m_N m_W}{m_H^2 - q^2} \tag{44}$$

where m_N is the nucleon mass, and the parameters m_h, m_H and α depend on the SUSY parameter space (see Table 1). If one ignores quarks other than u and d (model A) and uses $m_u = 5MeV = m_d/2$ finds [24]

$$f_s^0 = 1.86(g_u + g_d)/2, \quad f_s^1 = 0.49(g_u - g_d)/2, \tag{45}$$

4 Going from the Quark to the Nucleon Level

As we have already mentioned, one has to be a bit more careful in handling quarks other than u and d since their couplings are proportional to their mass [14]. One encounters in the nucleon not only sea quarks ($u\bar{u}, d\bar{d}$ and $s\bar{s}$) but the heavier quarks also due to QCD effects [15]. This way one obtains the scalar Higgs-nucleon coupling by using effective quark masses as follows

$$m_u \to f_u m_N, \quad m_d \to f_d m_N. \quad m_s \to f_s m_N$$

$$m_Q \to f_Q m_N, \quad (heavy \;\; quarks \;\; c, b, t)$$

where m_N is the nucleon mass. The isovector contribution is now negligible. The parameters f_q, $q = u, d, s$ can be obtained by chirala symmetry breaking terms in relation to phase shift and dispersion analysis. Following Cheng and Cheng [25] we obtain

$$f_u = 0.021, \quad f_d = 0.037, \quad f_s = 0.140 \qquad \text{(model B)}$$

$$f_u = 0.023, \quad f_d = 0.034, \quad f_s = 0.400 \qquad \text{(model C)}$$

We see that in both models the s-quark is dominant. Then to leading order via quark loops and gluon exchange with the nucleon one finds:

$$f_Q = 2/27(1 - \quad \sum_q f_q)$$

This yields:

$$f_Q = 0.060 \qquad \text{(model C)}$$

$$f_Q = 0.040 \qquad \text{(model C)}$$

There is a correction to the above parameters coming from loops involving s-quarks [15]. The leading contribution can be absorbed into the definition if the functions $g_1(h)$ and $g_2(H)$ as follows :

$$g_1(h) \to g_1(h)[1 + \tfrac{1}{8}(2\tfrac{m_Q^2}{m_W^2} - \tfrac{sin(\alpha+\beta)}{cos^2\theta_W}\tfrac{sin\beta}{cos\alpha})]$$

$$g_2(H) \to g_1(h)[1 + \tfrac{1}{8}(2\tfrac{m_Q^2}{m_W^2} + \tfrac{cos(\alpha+\beta)}{cos^2\theta_W}\tfrac{sin\beta}{sin\alpha})]$$

for $Q = c$ and t For the b-quark we get:

$$g_1(h) \to g_1(h)[1 + \tfrac{1}{8}(2\tfrac{m_b^2}{m_W^2} - \tfrac{sin(\alpha+\beta)}{cos^2\theta_W}\tfrac{cos\beta}{cos\alpha})]$$

$$g_2(H) \to g_1(h)[1 + \tfrac{1}{8}(2\tfrac{m_b^2}{m_W^2} - \tfrac{cos(\alpha+\beta)}{cos^2\theta_W}\tfrac{cos\beta}{sin\alpha})]$$

In addition to the above effects one has to consider QCD effects. These effects renormalize the quark loops as follows [15]:

$$f_{QCD}(q) = \tfrac{1}{4}\tfrac{\beta(\alpha_s)}{1+\gamma_m(\alpha_s)}$$

with

$$\beta(\alpha_s) = \tfrac{\alpha_s}{3\pi}[1 + \tfrac{19}{4}\alpha_s\pi] \ , \ \gamma_m(\alpha_s) = 2\tfrac{\alpha_s}{\pi}$$

Thus

$$f_{QCD}(q) = 1 + \tfrac{11}{4}\tfrac{\alpha_s}{\pi}$$

The QCD correction associated with the s-quark loops is:

$$f_{QCD}(\tilde{q}) = 1 + \tfrac{25}{6}\tfrac{\alpha_s}{\pi}$$

The above corrections depend on Q since α_s must be evaluated at the scale of m_Q.

It convenient to introduce the factor$f_{QCD}(\tilde{q})/f_{QCD}(q)$ into the factors $g_1(h)$ and $g_2(H)$ and the factor of $f_{QCD}(q)$ into the the quantities f_Q. If, however, one restricts oneself to the large $tan\beta$ regime, the corrections due to the s-quark loops is independent of the parameters α and β and significant only for the t-quark.

For large $tan\beta$ we find:

$$f_c = 0.060 \times 1.068 = 0.064, f_t = 0.060 \times 2.048 = 0.123, f_b = 0.060 \times 1.174 = 0.070 \text{ quad (model B)}$$

$$f_c = 0.040 \times 1.068 = 0.043, f_t = 0.040 \times 2.048 = 0.082, f_b = 0.040 \times 1.174 = 0.047 \text{ quad (model B)}$$

For a more detailed discussion we refer the reader to Refs. [14, 15]

Table 2: The coupling constants entering $\mathcal{L}_{eff}$, Eqs. (46), (47) and (49) of the text, for solutions #1 - #3.

Quantity	Solution #1	Solution #2	Solution #3
βf_V^0	1.746×10^{-5}	2.617×10^{-5}	2.864×10^{-5}
f_V^1/f_V^0	-0.153	-0.113	-0.251
$f_S^0(H)$ (model A)	1.31×10^{-5}	1.30×10^{-4}	1.38×10^{-5}
f_S^1/f_S^0 (model A)	-0.275	-0.107	-0.246
$f_S^0(H)$ (model B)	5.29×10^{-4}	7.84×10^{-3}	6.28×10^{-4}
$f_S^0(H)$ (model C)	7.57×10^{-4}	7.44×10^{-3}	7.94×10^{-4}
$f_A^0(NQM)$	0.510×10^{-2}	3.55×10^{-2}	0.704×10^{-2}
$f_A^0(EMC)$	0.612×10^{-3}	0.426×10^{-2}	0.844×10^{-3}
f_A^1	1.55×10^{-2}	5.31×10^{-2}	3.00×10^{-2}

5 Summary of the Input Parameters.

We have seen that, the vector and axial vector form factors can arise out of Z-exchange and s-quark exchange. [12]-[16] They have uncertainties in them. Here we consider the three choices in the allowed parameter space of Kane *et al* [21] and the eight parameter choices of Castano *et al* [21] These involve universal soft breaking masses at the scale. Non-universal masses have also recently been employed [14] (see also Arnowitt and Nath [22] In our choice of the parameters the LSP is mostly a gaugino. Thus, the Z- contribution is small. It may become dominant in models in which the LSP happens to be primarily a Higgsino. Such models, however, are excluded by the cosmological bounds on the relic abundance of LSP. The transition from the quark to the nucleon level is pretty straightforward in the case of vector current contribution. We will see later that, due to the Majorana nature of the LSP, the contribution of the vector current, which can lead to a coherent effect of all nucleons, is suppressed. [12] The vector current is effectively multiplied by a factor of $\beta = v/c$, v is the velocity of LSP (see Tables 2, 3). Thus, the axial current, especially in the case of light and medium mass nuclei, cannot be ignored.

For the isovector axial current one is pretty confident about how to go from the quark to the nucleon level. We know from ordinary weak decays that the coupling merely gets renormalized from $g_A = 1$ to $g_A = 1.24$. For the isoscalar axial current the situation is not completely clear. The naive quark model (NQM) would give a renormalization parameter of unity (the same as the isovector vector current). This point of view has, however, changed in recent years due to the so-called spin crisis, [40]-[42] i.e. the fact that in the EMC data [40] it appears that only a small fraction of the proton spin arises from the quarks. Thus, one may have to renormalize f_A^0 by $g_A^0 = 0.28$, for u and d quarks, and $g_A^0 = -0.16$ for the strange quarks, [41, 42] i.e. a total factor of 0.12. These two possibilities, labeled as NQM and EMC, are listed in Tables 2,3. One cannot completely rule out the possibility that the actual value maybe anywhere in the above mentioned region. [42]

The scalar form factors arise out of the Higgs exchange or via s-quark exchange when there is mixing [14] between s-quarks $\tilde{q}_L$ and $\tilde{q}_R$ (the partners of the left-

Table 3: The same as in Table 2 for solutions #4 - #9.

Solution	#4	#5	#6	#7	#8	#9
$\langle\beta^2\rangle^{1/2} f_V^0$	$0.225\ 10^{-4}$	$0.190\ 10^{-4}$	$0.358\ 10^{-4}$	$0.108\ 10^{-4}$	$0.694\ 10^{-4}$	$0.864\ 10^{-4}$
f_V^1/f_V^0	-0.0809	-0.0050	-0.0320	-0.0538	-0.0464	-0.0369
$f_S^0(A)$	$-0.179\ 10^{-4}$	$-0.236\ 10^{-4}$	$-0.453\ 10^{-4}$	$-0.266\ 10^{-4}$	$-0.210\ 10^{-3}$	$-0.131\ 10^{-3}$
$f_S^0(B)$	$-0.531\ 10^{-2}$	$-0.145\ 10^{-2}$	$-0.281\ 10^{-2}$	$-0.132\ 10^{-1}$	$-0.117\ 10^{-1}$	$-0.490\ 10^{-2}$
$f_S^0(C)$	$-0.315\ 10^{-2}$	$-0.134\ 10^{-2}$	$-0.261\ 10^{-2}$	$-0.153\ 10^{-1}$	$-0.118\ 10^{-1}$	$-0.159\ 10^{-2}$
$f_S^1(A)$	$-0.207\ 10^{-5}$	$-0.407\ 10^{-5}$	$0.116\ 10^{-4}$	$0.550\ 10^{-4}$	$0.307\ 10^{-4}$	$0.365\ 10^{-4}$
$f_A^0(NQM)$	$6.950\ 10^{-3}$	$5.800\ 10^{-3}$	$1.220\ 10^{-2}$	$3.760\ 10^{-2}$	$3.410\ 10^{-2}$	$2.360\ 10^{-2}$
$f_A^0(EMC)$	$0.834\ 10^{-3}$	$0.696\ 10^{-3}$	$0.146\ 10^{-2}$	$0.451\ 10^{-2}$	$0.409\ 10^{-2}$	$0.283\ 10^{-2}$
f_A^1	$2.490\ 10^{-2}$	$1.700\ 10^{-2}$	$3.440\ 10^{-2}$	$2.790\ 10^{-1}$	$1.800\ 10^{-1}$	$2.100\ 10^{-1}$

handed and right-handed quarks). We have seen [12] that they have two types of uncertainties in them. One, which is the most important, at the quark level due to the uncertainties in the Higgs sector. The actual values of the parameters f_S^0 and f_S^1 used here, arising mainly from Higgs exchange, were obtained by considering 1-loop corrections in the Higgs sector. As a result, the lightest Higgs mass is now a bit higher, i.e. more massive than the value of the Z-boson. [43, 44]

The other type of uncertainty is related to the step going from the quark to the nucleon level [14] (see sect. 3.3). Such couplings are proportional to the quark masses, and hence sensitive to the small admixtures of $q\bar{q}$ (q other than u and d) present in the nucleon. Again values of f_S^0 and f_S^1 in the allowed SUSY parameter space are considered (see Tables 2,3).

6 Expressions for the Unconvoluted Event Rates.

Combining for results of the previous section we can write

$$L_{eff} = -\frac{G_F}{\sqrt{2}}\{(\bar{\chi}_1\gamma^\lambda\gamma_5\chi_1)J_\lambda + (\bar{\chi}_1\chi_1)J\} \tag{46}$$

where

$$J_\lambda = \bar{N}\gamma_\lambda(f_V^0 + f_V^1\tau_3 + f_A^0\gamma_5 + f_A^1\gamma_5\tau_3)N \tag{47}$$

with

$$\begin{aligned} f_V^0 &= f_V^0(Z) + f_V^0(\tilde{q}), \qquad f_V^1 = f_V^1(Z) + f_V^1(\tilde{q}) \\ f_A^0 &= f_A^0(Z) + f_A^0(\tilde{q}), \qquad f_A^1 = f_A^1(Z) + f_A^1(\tilde{q}) \end{aligned} \tag{48}$$

and

$$J = \bar{N}(f_s^0 + f_s^1\tau_3)N \tag{49}$$

We have neglected the uninteresting pseudoscalar and tensor currents. Note that, due to the Majorana nature of the LSP, $\bar{\chi}_1\gamma^\lambda\chi_1 = 0$ (identically).

With the above ingredients the differential cross section can be cast in the form

$$d\sigma(u,v) = \frac{du}{2(\mu_r bv)^2}[(\bar{\Sigma}_S + \bar{\Sigma}_V \ \frac{v^2}{c^2}) \ F^2(u) + \bar{\Sigma}_{spin}F_{11}(u)] \tag{50}$$

$$\bar{\Sigma}_S = \sigma_0(\frac{\mu_r}{m_N})^2 \, \{A^2 \, [(f_S^0 - f_S^1\frac{A-2Z}{A})^2] \simeq \sigma^S_{p,\chi^0} A^2(\frac{\mu_r}{m_N})^2 \tag{51}$$

$$\bar{\Sigma}_{spin} = \sigma^{spin}_{p,\chi^0} \ \zeta_{spin} \tag{52}$$

$$\zeta_{spin} = \frac{(\mu_r/m_N)^2}{3(1+\frac{f_A^0}{f_A^1})^2}[(\frac{f_A^0}{f_A^1}\Omega_0(0))^2\frac{F_{00}(u)}{F_{11}(u)} + 2\frac{f_A^0}{f_A^1}\Omega_0(0)\Omega_1(0)\frac{F_{01}(u)}{F_{11}(u)} + \Omega_1(0))^2\,] \tag{53}$$

$$\bar{\Sigma}_V = \sigma^V_{p,\chi^0} \ \zeta_V \tag{54}$$

$$\zeta_V = \frac{(\mu_r/m_N)^2}{(1+\frac{f_V^1}{f_V^0})^2}A^2\,(1-\frac{f_V^1}{f_V^0}\ \frac{A-2Z}{A})^2[(\frac{v_0}{c})^2[1-\frac{1}{(2\mu_r b)^2}\frac{2\eta+1}{(1+\eta)^2}\frac{\langle\ 2u\ \rangle}{\langle\ v^2\ \rangle}] \tag{55}$$

σ^i_{p,χ^0} = proton cross-section,$i = S, spin, V$ given by:
$\sigma^S_{p,\chi^0} = \sigma_0 \ (f_S^0)^2$ (scalar) , (the isovector scalar is negligible, i.e. $\sigma_p^S = \sigma_n^S$)
$\sigma^{spin}_{p,\chi^0} = \sigma_0 \ \ 3 \ (f_A^0 + f_A^1)^2$ (spin) , $\sigma^V_{p,\chi^0} = \sigma_0 \ (f_V^0 + f_V^1)^2$ (vector)
where m_p is the proton mass, $\eta = m_x/m_N A$, and μ_r is the reduced mass and

$$\sigma_0 = \frac{1}{2\pi}(G_F m_N)^2 \simeq 0.77 \times 10^{-38} cm^2 \tag{56}$$

$$u = q^2 b^2/2 \tag{57}$$

or equivalently

$$Q = Q_0 u, \qquad Q_0 = \frac{1}{A m_N b^2} \tag{58}$$

where b is (the harmonic oscillator) size parameter, q is the momentum transfer to the nucleus, and Q is the energy transfer to the nucleus (see Table 4)
In the above expressions $F(u)$ is the nuclear form factor and

$$F_{\rho\rho'}(u) = \sum_{\lambda,\kappa} \frac{\Omega_\rho^{(\lambda,\kappa)}(u)}{\Omega_\rho(0)}\ \frac{\Omega_{\rho'}^{(\lambda,\kappa)}(u)}{\Omega_{\rho'}(0)}, \qquad \rho,\rho' = 0,1 \tag{59}$$

are the spin form factors [13] (ρ, ρ' are isospin indices)
Both form factors are normalized to one at $u = 0$.
Ω_0 (Ω_1) are the static isoscalar (isovector) spin matrix elements (see tables 5 and 6).

The non-directional event rate is given by:

$$R = R_{non-dir} = \frac{dN}{dt} = \frac{\rho(0)}{m_\chi}\frac{m}{A m_N}\sigma(u,v)|v| \tag{60}$$

Where $\rho(0) = 0.3 GeV/cm^3$ is the LSP density in our vicinity and m is the detector mass

Table 4: The quantity q_0 (forward momentum transfer) in units of fm^{-1} for three values of m_χ and three typical nuclei. In determining q_0 the value $\langle\beta^2\rangle^{1/2} = 10^{-3}$ was employed.

	q_0 (fm^{-1})		
Nucleus	$m_\chi = 30.0\,GeV$	$m_\chi = 100.0\,GeV$	$m_\chi = 150.0\,GeV$
Ca	0.174	0.290	0.321
Ge	0.215	0.425	0.494
Pb	0.267	0.685	0.885

Table 5: Comparison of the static spin matrix elements for three typical nuclei, Pb (present calculation) and ^{73}Ge, ^{19}F , ^{23}Na, ^{29}Si (see Ref. [26, 30]).

Component	$^{207}Pb_{1/2^-}$	$^{73}Ge_{9/2^+}$	$^{19}Si_{1/2^+}$	$^{23}Na_{3/2^+}$	$^{29}Si_{1/2^+}$
$\Omega_1^2(0)$	0.231	1.005	2.807	0.346	0.220
$\Omega_1(0)\Omega_0(0)$	-0.266	-1.078	2.707	0.406	-0.214
$\Omega_0^2(0)$	0.305	1.157	2.610	0.478	0.208

The differential non-directional rate can be written as

$$dR = dR_{non-dir} = \frac{\rho(0)}{m_\chi}\frac{m}{Am_N}d\sigma(u,v)|v| \tag{61}$$

where $d\sigma(u,v)$ was given above.

The directional differential rate [45] in the direction $\hat{e}$ is given by :

$$dR_{dir} = \frac{\rho(0)}{m_\chi}\frac{m}{Am_N}v.\hat{e}H(v.\hat{e})\ \frac{1}{2\pi}\ d\sigma(u,v) \tag{62}$$

where H the Heaviside step function. The factor of $1/2\pi$ is introduced, since the differential cross section of the last equation is the same with that entering the non-directional rate, i.e. after an integration over the azimuthal angle around the nuclear momentum has been performed. In other words, crudely speaking, $1/(2\pi)$ is the suppression factor we expect in the directional rate compared to the usual one. The precise suppression factor depends, of course, on the direction of observation.

7 Convolution of the Event Rates.

We have seen that the event rate for LSP-nucleus scattering depends on the relative LSP-target velocity. In this section we will examine the consequences of the earth's revolution around the sun (the effect of its rotation around its axis is expected to

Table 6: Ratio of spin contribution ($^{207}Pb/^{73}Ge$) at the relevant momentum transfer with the kinematical factor $1/(1+\eta)^2$, $\eta = m/Am_N$.

Solution	#1	#2	#3	#4	#5	#6	#7	#8	#9
$m_x\,(GeV)$	126	27	102	80	124	58	34	35	50
NQM	0.834	0.335	0.589	0.394	0.537	0.365	0.346	0.337	0.417
EMC	0.645	0.345	0.602	0.499	0.602	0.263	0.341	0.383	0.479

be negligible) i.e. the modulation effect. In practice this has been accomplished by assuming a consistent LSP velocity dispersion, such as a Maxwell distribution [2]. More recently other non-isothermal approaches, in the context velocity peaks and caustic rings, have been proposed, see e.g Sikivie et al [31]. Let us begin with isothermal models. In the present paper following the work of Drukier, see Ref. [32], we will assume that the velocity distribution is only axially symmetric, i.e. of the form

$$f(v',\lambda) = N(y_{esc},\lambda)(\sqrt{\pi}v_0)^{-3})[f_1(v',\lambda) - f_2(v',v_{esc},\lambda)] \tag{63}$$

with

$$f_1(v',\lambda) = exp[(-\frac{(v_x')^2 + (1+\lambda)((v_y')^2 + (v_z')^2)}{v_0^2}] \tag{64}$$

$$f_2(v',v_{esc},\lambda) = exp[-\frac{v_{esc}^2 + \lambda((v_y')^2 + (v_z')^2)}{v_0^2}] \tag{65}$$

where

$$v_0 = \sqrt{(2/3)\langle v^2\rangle} = 220Km/s \tag{66}$$

i.e. v_0 is the velocity of the sun around the center of the galaxy. v_{esc} is the escape velocity in the gravitational field of the galaxy, $v_{esc} = 625Km/s$ [32]. In the above expressions λ is a parameter, which describes the asymmetry and takes values between 0 and 1 and N is a proper normalization constant [18]. For $y_{esc} \to \infty$ we get the simple expression $N^{-1} = \lambda + 1$

In a recently proposed non-isothermal model one consider the late in-fall of dark matter into our galaxy producing flows of caustic rings. In particular the predictions of a self-similar model have been put forward as a possible scenario for dark matter density-velocity distribution, see e.g. Sikivie et al [31]. The implications of such theoretical predictions and, in particular, the modulation effect are the subject of this section.

Following Sikivie we will consider $2\times N$ caustic rings, (i,n) , i=(+.-) and n=1,2,...N (N=20 in the model of Sikivie et al), each of which contributes to the local density a fraction $\bar{\rho}_n$ of the of the summed density $\bar{\rho}$ of each of the $i = +,-$. It contains WIMP like particles with velocity $\mathbf{y}_n' = (y_{nx}', y_{ny}', y_{nz}')$ in units of essentially the sun's velocity ($v_0 = 220\ Km/s$), with respect to the galactic center. The z-axis is chosen in the direction of the disc's rotation, i.e. in the direction of the motion of the the sun, the y-axis is perpendicular to the plane of the galaxy and the x-axis

Table 7: The velocity parameters $y_n^{'} = \upsilon_n/\upsilon_0$, $y_{nz} = y_{nz}^{'} = \upsilon_{n\phi}/\upsilon_0$, $y_{ny} = y_{ny}^{'} = \upsilon_{nz}/\upsilon_0$, $y_{nx} = y_{nx}^{'} = \upsilon_{nr}/\upsilon_0$ and $y_n = [(y_{nz}-1)^2 + y_{ny}^2 + y_{nx}^2]^{1/2}$. Also shown are the quantities: a_n, the caustic rind radii, and $\bar{\rho}_n = d_n/[\sum_{n=1}^{20} d_n]$. (For the other definitions see text).

n	$a_n(Kpc)$	$y_n^{'}$	y_{nz}	y_{ny}	y_{nx}	y_n	$\bar{\rho}_n$
1	38.0	2.818	0.636	±2.750	0.000	2.773	0.0120
2	19.0	2.568	1.159	±2.295	0.000	2.301	0.0301
3	13.0	2.409	1.591	±1.773	0.000	1.869	0.0601
4	9.7	2.273	2.000	±1.091	0.000	1.480	0.1895
5	7.8	2.182	2.000	0.000	±0.863	1.321	0.2767
6	6.5	2.091	1.614	0.000	±1.341	1.475	0.0872
7	5.6	2.023	1.318	0.000	±1.500	1.533	0.0571
8	4.9	1.955	1.136	0.000	±1.591	1.597	0.0421
9	4.4	1.886	0.977	0.000	±1.614	1.614	0.0331
10	4.0	1.818	0.864	0.000	±1.614	1.619	0.0300
11	3.6	1.723	0.773	0.000	±1.614	1.630	0.0271
12	3.3	1.723	0.682	0.000	±1.591	1.622	0.0241
13	3.1	1.619	0.614	0.000	±1.568	1.615	0.0211
14	2.9	1.636	0.545	0.000	±1.545	1.611	0.0180
15	2.7	1.591	0.500	0.000	±1.500	1.581	0.0180
16	2.5	1.545	0.454	0.000	±1.477	1.575	0.0165
17	2.4	1.500	0.409	0.000	±1.454	1.570	0.0150
18	2.2	1.455	0.386	0.000	±1.409	1.537	0.0150
19	2.1	1.432	0.364	0.000	±1.386	1.525	0.0135
20	2.0	1.409	0.341	0.000	±1.364	1.515	0.0135

is in the radial direction. We caution the reader that these axes are traditionally indicated by astronomers as $\hat{e}_\phi, \hat{e}_r, \hat{e}_z$) respectively. The needed quantities are taken from the work of Sikivie et al [31] (see Table 7), via the definitions

$y_n^{'} = \upsilon_n/\upsilon_0, y_{nz}^{'} = \upsilon_{n\phi}/\upsilon_0 = y_{nz}, y_{nx}^{'} = \upsilon_{nr}/\upsilon_0 = y_{nx}, y_{ny}^{'} = \upsilon_{nz}/\upsilon_0 = y_{ny}, \rho_n = d_n/\bar{\rho}, \bar{\rho} = \sum_{n=1}^{N} d_n$ and $y_n = [(y_{nz}-1)^2 + y_{ny}^2 + y_{nx}^2]^{1/2}$ (for each flow +.-). This leads to a velocity distribution of the form:

$$f(\upsilon') = \sum_{n=1}^{N} \delta(\upsilon^{'} - \upsilon_0 \, \mathbf{y}_n^{'}) \tag{67}$$

The actual situation, of course, could be a combination of an isothermal contribution and late infall of dark matter [45]. In the present treatment we will consider each of these distributions separately. For clarity of presentation we will consider each case separately.

Since the axis of the ecliptic [13]. lies very close to the y, z plane the velocity of the earth around the sun is given by

$$\upsilon_E \;=\; \upsilon_0 \;+\; \upsilon_1 \;=\; \upsilon_0 + \upsilon_1(\,sin\alpha\,\hat{\mathbf{x}} - cos\alpha\,cos\gamma\,\hat{\mathbf{y}} + cos\alpha\,sin\gamma\,\hat{\mathbf{z}}\,) \tag{68}$$

where α is the phase of the earth's orbital motion, $\alpha = 2\pi(t-t_1)/T_E$, where t_1 is around second of June and $T_E = 1year$.

One can now express the above distribution in the laboratory frame [18] by writing $v' = v + v_E$

8 Expressions for the Non-directional Differential Event Rate

The mean value of the non-directional event rate of Eq. (61), is given by

$$\left\langle \frac{dR}{du} \right\rangle = \frac{\rho(0)}{m_\chi} \frac{m}{Am_N} \int f(v, v_E)|v| \frac{d\sigma(u,v)}{du} d^3v \tag{69}$$

The above expression can be more conveniently written as

$$\left\langle \frac{dR}{du} \right\rangle = \frac{\rho(0)}{m_\chi} \frac{m}{Am_N} \sqrt{\langle v^2 \rangle} \langle \frac{d\Sigma}{du} \rangle \tag{70}$$

where

$$\langle \frac{d\Sigma}{du} \rangle = \int \frac{|v|}{\sqrt{\langle v^2 \rangle}} f(v, v_E) \frac{d\sigma(u,v)}{du} d^3v \tag{71}$$

8.1 No velocity Dispersion-The Case of Caustic Rings

In the case of caustic rings the last expression takes the form

$$\langle \frac{d\Sigma}{du} \rangle = \frac{2\bar{\rho}}{\rho(0)} \, a^2 [\bar{\Sigma}_S \bar{F}_0(u) + \frac{\langle v^2 \rangle}{c^2} \bar{\Sigma}_V \bar{F}_1(u) + \bar{\Sigma}_{spin} \bar{F}_{spin}(u)] \tag{72}$$

We remind the reader that $\bar{\rho}$ was obtain for each type of flow (+ or -), which explains the factor of two. In the Sikivie model [31] we have $(2\bar{\rho}/\rho(0) = 1.25$, i.e. the whole dark matter density lies in the form of caustic rings. In a composite model this can only be a fraction of the total density.

The quantities $\bar{\Sigma}_i, i = S, V, spin$ are given by Eqs. (51)- (54). The quantities $\bar{F}_0, \bar{F}_1, \bar{F}_{spin}$ are obtained from the corresponding form factors via the equations

$$\bar{F}_k(u) = F^2(u) \bar{\Psi}_k(u) \frac{(1+k)}{2k+1} \quad , \quad k = 0, 1 \tag{73}$$

$$\bar{F}_{spin}(u) = F_{11}(u) \bar{\Psi}_0(u) \tag{74}$$

The functions $\tilde{\Psi}_k(u)$ depend on the model. Introducing the parameter

$$\delta = \frac{2v_1}{v_0} = 0.27, \tag{75}$$

in the Sikivie model we find

$$\begin{aligned} \tilde{\Psi}_k(u) &= \sqrt{\frac{2}{3}} \sum_{n=1}^{N} \bar{\rho}_n y_n^{2(k-1)} \Theta(\frac{y_n^2}{a^2} - u) [(y_{nz} - 1 - \frac{\delta}{2} \, sin\gamma \, cos\alpha)^2 \\ &+ (y_{ny} + \frac{\delta}{2} \, cos\gamma \, cos\alpha)^2 + (y_{nx} - \frac{\delta}{2} \, sin\alpha)^2]^{1/2} \end{aligned} \tag{76}$$

with

$$a = \frac{1}{\sqrt{2}\mu_r b v_0} \tag{77}$$

Combining the above results the non-directional differential rate takes the form

$$\langle\frac{dR}{du}\rangle = \bar{R}\frac{2\bar{\rho}}{\rho(0)}t\ T(u)[1 - \cos\alpha\ H(u)] \tag{78}$$

In the above expressions $\bar{R}$ is the rate obtained in the conventional approach [12] by neglecting the folding with the LSP velocity and the momentum transfer dependence of the differential cross section, i.e. by

$$\bar{R} = \frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\sqrt{\langle v^2\rangle}[\bar{\Sigma}_S + \bar{\Sigma}_{spin} + \frac{\langle v^2\rangle}{c^2}\bar{\Sigma}_V] \tag{79}$$

where $\bar{\Sigma}_i, i = S, V, spin$ have been defined above, see Eqs (51) - (54).

The factor $T(u)$ takes care of the u-dependence of the unmodulated differential rate. It is defined so that

$$\int_{u_{min}}^{u_{max}} duT(u) = 1. \tag{80}$$

i.e. it is the relative differential rate. u_{min} is determined by the energy cutoff due to the performance of the detector,i.e

$$u_{min} = \frac{Q_{min}}{Q_0} \tag{81}$$

while u_{max} is determined the via the relations:

$$u_{max} = min(\frac{y_{esc}^2}{a^2}, max(\frac{y_n^2}{a^2})\ ,\ n = 1, 2, ..., N) \tag{82}$$

On the other hand $H(u)$ gives the energy transfer dependent modulation amplitude (relative to the unmodulated amplitude). The quantity t takes care of the modification of the total rate due to the nuclear form factor and the folding with the LSP velocity distribution. Since the functions $\bar{F}_0(u), \bar{F}_1$ and $\bar{F}_{spin}$ have a different dependence on u, the functions $T(u)$and $H(u)$ and t, in principle, depend somewhat on the SUSY parameters. If, however, we ignore the small vector contribution and assume (i) the scalar and axial (spin) dependence on u is the same, as seems to be the case for light systems [30]'[39], or (ii) only one mechanism (S, V, spin) dominates, the parameter $\bar{R}$ contains the dependence on all SUSY parameters. The parameters t and $T(u)$ depend on the LSP mass and the nuclear parameters, while the $H(u)$ depends only on the parameter a.

8.2 Velocity Dispersion-Isothermal Models

Expanding in powers of δ , see Eq. (75) and keeping terms up to linear in it we can manage to perform the angular integrations [18] in Eq. (71) and get Eq. (72). Now the quantities $\tilde{\Psi}_k(u)$ are given by

$$\tilde{\Psi}_k(u) = [\tilde{\psi}_{(0),k}(a\sqrt{u}) + 0.135\cos\alpha\tilde{\psi}_{(1),k}(a\sqrt{u})] \tag{83}$$

and

$$\tilde{\psi}_{(l),k}(x) = N(y_{esc},\lambda)e^{-\lambda}(e^{-1}\tilde{\Phi}_{(l),k}(x) - exp[-y^2_{esc}]\tilde{\Phi}'_{(l),k}(x)) \quad (84)$$

$$\tilde{\Phi}_{(l),k}(x) = \frac{2}{\sqrt{6\pi}}\int_x^{y_{esc}} dyy^{2k-1}exp(-(1+\lambda)y^2))(\tilde{F}_l(\lambda,(\lambda+1)2y) + \tilde{G}_l(\lambda,y))) \quad (85)$$

$$\tilde{\Phi}'_{(l),k}(x) = \frac{2}{\sqrt{6\pi}}\int_x^{y_{esc}} dyy^{2k-1}exp(-\lambda y^2))\tilde{G}'_l(\lambda,y)) \quad (86)$$

In the above expressions

$$\tilde{G}_0(0,y) = 0 \quad , \quad \tilde{G}_1(0,y) = 0 \quad (87)$$

$$\tilde{F}_0(\lambda,x) = (\lambda+1)^{-2}\ x\ sinh(x) \quad (88)$$

$$\tilde{F}_1(\lambda,x) = (1+\lambda)^{-2}\ [(2+\lambda)(x/2)+1)\ [x\ cosh(x) - (2\lambda+3)\ sinh(x)] \quad (89)$$

note that here $x = (\lambda+1)2y$. The functions $\tilde{G}$ cannot be obtained analytically, but they can easily be expressed as a rapidly convergent series in $y = \frac{v}{v_0}$, which will not be given here.

The functions $\tilde{G}'_i(\lambda,y)$, associated with the small second term of the velocity distribution are obtained similarly [18].

The non-directional differential rate takes the form

$$\langle\frac{dR}{du}\rangle = \bar{R}\frac{\rho'(0)}{\rho(0)}tT(u)[(1+\cos\alpha H(u))] \quad (90)$$

With $\bar{R}$ given by Eq. (79) and $\rho'(0)$ is the part of the total LSP density attributed to this mode. Note the difference of sign in the definition of the modulation amplitude H compared to Eq. (78).

Here u_{min} is determined by the energy cutoff due to the performance of the detector. u_{max} is determined by the escape velocity v_{esc} via the relation:

$$u_{max} = \frac{y^2_{esc}}{a^2} \quad (91)$$

Considering only the scalar interaction we get $\bar{R} \to \bar{R}_S$ and

$$t\ T(u) = a^2F^2(u)\tilde{\psi}_{(0),0}(a\sqrt{u}) \quad (92)$$

For the spin interaction we get a similar expression except that $\bar{R} \to \bar{R}_{spin}$ and $F^2 \to F_{11}$. Finally for completeness we will consider the less important vector contribution. We get $\bar{R} \to \bar{R}_V$ and

$$t\ T(u) = F^2(u)[\tilde{\psi}_{(0),1}(a\sqrt{u}) - \frac{1}{(2\mu_r b)^2}\frac{2\eta+1}{(1+\eta)^2}u\,\tilde{\psi}_{(0),0}(a\sqrt{u})]\frac{2a^2}{3} \quad (93)$$

The quantity $T(u)$ depends on the nucleus through the nuclear form factor or the spin response function and the parameter a. The modulation amplitude takes the form

$$H(u) = 0.135\frac{\tilde{\psi}_{(1),k}(a\sqrt{u})}{\tilde{\psi}_{(0),k}(a\sqrt{u})} \quad (94)$$

Thus in this case the $H(u)$ depends only on $a\sqrt{u}$, which coincides with the parameter x of Ref. [35], i.e. only on the momentum transfer, the reduced mass and the size of the nucleus.

Returning to the differential rate it is sometimes convenient to use the quantity $T(u)H(u)$ rather than H, since H(u) may appear artificially increasing function of u due to the faster decrease of $T(u)$ (H(u) was obtained after division by T(u))

9 Expressions for the Directional Differential Event Rate

There are now experiments under way aiming at measuring directional rates [38] using TPC counters which permit the observation of the recoiling nucleus is observed in a certain direction. From a theoretical point of view the directional rates have been previously discussed by Spegel [37] and Copi et al [45]. The rate will depend on the direction of observation, showing a strong correlation with the direction of the sun's motion. In a favorable situation the rate will merely be suppressed by a factor of 2π relative to the non-directional rate. This is due to the fact that one does not now integrate over the azimuthal angle of the nuclear recoiling momentum. The directional rate will also show modulation due to the Earth's motion. We will again examine an non-isothermal non symmetric case (the Sikivie model) and a 3-dimensional Gaussian distribution with only axial symmetry.

The mean value of the directional differential event rate of Eq. (62), is defined by

$$\left\langle \frac{dR}{du} \right\rangle_{dir} = \frac{\rho(0)}{m_\chi} \frac{m}{Am_N} \frac{1}{2\pi} \int f(\upsilon, \upsilon_E) \upsilon.\hat{e} H(\upsilon.\hat{e}) \frac{d\sigma(u,\upsilon)}{du} d^3\upsilon \tag{95}$$

where $\hat{\mathbf{e}}$ is the unit vector in the direction of observation. It can be more conveniently expressed as

$$\left\langle \frac{dR}{du} \right\rangle_{dir} = \frac{\rho(0)}{m_\chi} \frac{m}{Am_N} \sqrt{\langle \upsilon^2 \rangle} \langle \frac{d\Sigma}{du} \rangle_{dir} \tag{96}$$

where

$$\langle \frac{d\Sigma}{du} \rangle_{dir} = \frac{1}{2\pi} \int \frac{\upsilon.\hat{e} H(\upsilon.\hat{e})}{\sqrt{\langle \upsilon^2 \rangle}} f(\upsilon, \upsilon_E) \frac{d\sigma(u,\upsilon)}{du} d^3\upsilon \tag{97}$$

9.1 Directional Differential Event Rate in the Case of Caustic Rings.

The model of Sikivie et al [31], which is a non isothermal and asymmetric one, offers itself as a perfect example for the study of directional rates. So we are going to begin our discussion with such a case. Working as in the previous section we get [18]

$$\langle \frac{d\Sigma}{du} \rangle_{dir} = \frac{2\bar{\rho}}{\rho(0)} \frac{a^2}{2\pi} [\bar{\Sigma}_S F_0(u) + \frac{\langle \upsilon^2 \rangle}{c^2} \bar{\Sigma}_V F_1(u) + \bar{\Sigma}_{spin} F_{spin}(u)] \tag{98}$$

where the $\bar{\Sigma}_i, i = S, V, spin$ are given by Eqs. (51)- (54). The quantities F_0, F_1, F_{spin} are obtained from the equations

$$F_k(u) = F^2(u) \Psi_k(u) \frac{(1+k)}{2k+1}, k = 0, 1 \tag{99}$$

$$F_{spin}(u) = F_{11}(u) \Psi_0(u) \tag{100}$$

In the Sikivie model we find

$$\begin{aligned} \Psi_k(u) &= \sqrt{\frac{2}{3}} \sum_{n=1}^{N} \bar{\rho}_n y_n^{2(k-1)} \Theta(\frac{y_n^2}{a^2} - u) |(y_{nz} - 1 - \frac{\delta}{2} \, sin\gamma \, cos\alpha) \mathbf{e}_z.\mathbf{e} \\ &+ (y_{ny} + \frac{\delta}{2} \, cos\gamma \, cos\alpha) \mathbf{e}_y.\mathbf{e} + (y_{nx} - \frac{\delta}{2} \, sin\alpha) \mathbf{e}_x.\mathbf{e}| \end{aligned} \tag{101}$$

In the model considered here the z-component of the LSP's velocity with respect to the galactic center for some rings is smaller than sun's velocity, while for some others it is larger. The components in the y and the x directions are opposite for the + and - flows. So we will distinguish the following cases: a) $\hat{e}$ has a component in the sun's direction of motion, i.e. $0 < \theta < \pi/2$, labeled by u (up). b) Detection in the direction specified by $\pi/2 < \theta < \pi$, labeled by d (down). The differential directional rate takes a different form depending on which quadrant the observation is made. Thus :

1. In the first quadrant (azimuthal angle $0 \leq \phi \leq \pi/2$).

$$
\begin{aligned}
\langle\frac{dR^i}{du}\rangle &= \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}T(u)[(R^i_z(u) - \cos\alpha\ H^i_1(u))|\mathbf{e}_z.\mathbf{e}I \\
&+ (R^i_y + cos\alpha H^i_2(u) + \frac{H^i_c(u)}{2}(|cos\alpha| + cos\alpha))|\mathbf{e}_y.\mathbf{e}| \\
&+ (R^i_x - sin\alpha H^i_3(u) + \frac{H^i_s(u)}{2}(|sin\alpha| - sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \qquad (102)
\end{aligned}
$$

2. In the second quadrant (azimuthal angle $\pi/2 \leq \phi \leq \pi$)

$$
\begin{aligned}
\langle\frac{dR^i}{du}\rangle &= \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}T(u)[(R^i_z(u) - \cos\alpha\ H^i_1(u))|\mathbf{e}_z.\mathbf{e}| \\
&+ (R^i_y + cos\alpha H^i_2(u) + \frac{H^i_c(u)}{2}(|cos\alpha| - cos\alpha))|\mathbf{e}_y.\mathbf{e}| \\
&+ (R^i_x + sin\alpha H^i_3(u) + \frac{H^i_s(u)}{2}(|sin\alpha| + sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \qquad (103)
\end{aligned}
$$

3. In the third quadrant (azimuthal angle $\pi \leq \phi \leq 3\pi/2$).

$$
\begin{aligned}
\langle\frac{dR^i}{du}\rangle &= \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}T(u)[(R^i_z(u) - \cos\alpha\ H^i_1(u))|\mathbf{e}_z.\mathbf{e}| \\
&+ (R^i_y - cos\alpha H^i_2(u) + \frac{H^i_c(u)}{2}(|cos\alpha| - cos\alpha))|\mathbf{e}_y.\mathbf{e}| \\
&+ (R^i_x + sin\alpha H^i_3(u) + \frac{H^i_s(u)}{2}(|sin\alpha| + sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \qquad (104)
\end{aligned}
$$

4. In the fourth quadrant (azimuthal angle $3\pi/2 \leq \phi \leq 2\pi$)

$$
\begin{aligned}
\langle\frac{dR^i}{du}\rangle &= \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}T(u)[(R^i_z(u) - \cos\alpha\ H^i_1(u))|\mathbf{e}_z.\mathbf{e}| \\
&+ (R^i_y - cos\alpha H^i_2(u) + \frac{H^i_c(u)}{2}(|cos\alpha| - cos\alpha))|\mathbf{e}_y.\mathbf{e}| \\
&+ (R^i_x - sin\alpha H^i_3(u) + \frac{H^i_s(u)}{2}(|sin\alpha| - sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \qquad (105)
\end{aligned}
$$

where $i = u, d$

By the reasoning given above, if one mechanism is dominant, the parameters $R_x, R_y, R_z, H_1, H_2, H_3, H_c, H_s$ for both directions u and d depend only on μ_r and a. They are all independent of the other SUSY parameters.

9.2 The Directional Differential Event Rate in the Case of Velocity Dispersion

The dependence of the rate depends on the direction of observation in a rather complicated way. The integrals can only be done numerically [45]. To simplify matters we made a power expansion in δ and kept terms up to linear in it. To make the presentation tractable we will will give expressions valid only for directions of greatest interest, i.e. close to the coordinate axes. In the sun's direction of motion we have a modulated as well as a non modulated amplitude. In the other two directions we only have a modulated amplitude. Unlike the case of caustic rings, now the direction opposite to the sun's direction of motion is favored. We found it more convenient, however, to present our results as the absolute value of the difference of the rates in the directions $\hat{e}$ and $-\hat{e}$.

Working as in the previous subsection we get [18]

$$\langle\frac{d\Sigma}{du}\rangle_{dir} = \frac{1}{2\pi}\, a^2[\bar{\Sigma}_S F_0(u) + \frac{\langle v^2\rangle}{c^2}\bar{\Sigma}_V F_1(u) + \bar{\Sigma}_{spin} F_{spin}(u)] \tag{106}$$

The quantities F_0, F_1, F_{spin} are obtained from the equations

$$F_k(u) = F^2(u)\Psi_k(u)\frac{(1+k)a^2}{2k+1}, k = 0,1 \tag{107}$$

Now

$$\begin{aligned}\Psi_k(u) &= \frac{1}{2}[(\psi_{(0),k}(a\sqrt{u}) + 0.135\cos\alpha\psi_{(1),k}(a\sqrt{u}))|\mathbf{e}_z.\mathbf{e}| \\ &- 0.117\cos\alpha\psi_{(2),k}(a\sqrt{u})|\mathbf{e}_y.\mathbf{e}| + 0.135\sin\alpha\psi_{(3),k}(a\sqrt{u})|\mathbf{e}_x.\mathbf{e}|]\end{aligned} \tag{108}$$

with

$$\psi_{(l),k}(x) = N(y_{esc},\lambda)e^{-\lambda}(e^{-1}\Phi_{(l),k}(x) - exp[-y_{esc}^2]\Phi'_{(l),k}(x)) \tag{109}$$

$$\Phi_{(l),k}(x) = \frac{2}{\sqrt{6\pi}}\int_x^{y_{esc}} dy y^{2k-1} exp(-(1+\lambda)y^2))(F_l(\lambda, 2(\lambda+1)y) + G_l(\lambda, y))) \tag{110}$$

$$\Phi'_{(l),k}(x) = \frac{2}{\sqrt{6\pi}}\int_x^{y_{esc}} dy y^{2k-1} exp(-\lambda y^2))G'_l(\lambda, y)) \tag{111}$$

In the above expressions

$$F_i(\lambda,\chi) = \chi cosh\chi - sinh\chi \quad , \quad i = 0,2,3 \tag{112}$$

$$F_1(\lambda,\chi) \;=\; 2(1-\lambda)\left[(\frac{(\lambda+1)\chi^2}{4(1-\lambda)} + 1)sinh\,\chi - \chi\,cosh\,\chi\right] \tag{113}$$

The purely asymmetric quantities G_i satisfy

$$G_i(0,y) = 0,\ i = 0,4 \tag{114}$$

the qualities $G_i'(0,y) = 0,\ i = 0,4$ refer to the second term of the Velocity distribution and were obtained in an analogous fashion.

If we consider each mode (scalar, spin vector) separately the directional rate takes the form

$$\langle\frac{dR}{du}\rangle_{dir} = \bar{R}\frac{\rho'(0)}{\rho(0)}\frac{t^0\ R^0}{4\pi}|(1+\cos\alpha H_1(u))\mathbf{e}_z.\mathbf{e} - \cos\alpha H_2(u)\mathbf{e}_y.\mathbf{e} + \sin\alpha H_3(u)\mathbf{e}_x.\mathbf{e}| \tag{115}$$

In other words the directional differential modulated amplitude is described in terms of the three parameters, $H_l(u)$, l=1,2 and 3. The unmodulated amplitude $R^0(u)$ is again normalized to unity. The parameter t^0 entering Eq. (115) takes care of whatever modifications are needed due to the convolution with the LSP velocity distribution in the presence of the nuclear form factors.

From Eqs. (106) - (115) we see that if we consider each mode separately the differential modulation amplitudes $H(l)$ take the form

$$H_l(u) = 0.135 \frac{\psi_k^{(l)}(a\sqrt{u})}{\psi_k^{(0)}(a\sqrt{u})} \quad , \quad l = 1,3 \quad ; \quad H_2(u) = 0.117 \frac{\psi_k^{(2)}(a\sqrt{u})}{\psi_k^{(0)}(a\sqrt{u})} \tag{116}$$

Thus in this case the H_l depend only on $a\sqrt{u}$, which coincides with the parameter x of Ref. [33]. This means that H_l essentially depend only on the momentum transfer, the reduced mass and the size of the nucleus. We note that in the case $\lambda = 0$ we have $H_2 = 0.117$ and $H_3 = 0.135$

It is sometimes convenient to use the quantity R_l rather than H_l defined by

$$R_l = R^0 H_l, \; l = 1, 2, 3. \tag{117}$$

The reason is that H_l, being the ratio of two quantities, may appear superficially large due to the denominator becoming small.

10 The Total Non-directional Event Rates

Integrating Eq. (78) we obtain for the total non-directional rate in the case of caustic rings the expression:

$$R = \bar{R} \; \frac{2\bar{\rho}}{\rho(0)} \; t \; [1 - h(a, Q_{min}) cos\alpha)] \tag{118}$$

where Q_{min} is the energy transfer cutoff imposed by the detector. The modulation is described by the parameter h. Similarly integrating Eq. (90) we obtain for the total non-directional rate in our isothermal model as follows:

$$R = \bar{R} \frac{\rho^{'}(0)}{\rho(0)} \; t \; [(1 + h(a, Q_{min}) cos\alpha)] \tag{119}$$

Note the difference of sign in the definition of the modulation amplitude h compared to Eq. (118). where Q_{min} is the energy transfer cutoff imposed by the detector. The modulation can be described in terms of the parameter h.

The effect of folding with LSP velocity on the total rate is taken into account via the quantity t. The SUSY parameters have been absorbed in $\bar{R}$. From our discussion in the case of differential rate it is clear that strictly speaking the quantities t and h also depend on the SUSY parameters. They do not depend on them, however, if one considers the scalar, spin etc. modes separately.

The meaning of t is clear from the above discussion. We only like to stress that it is a common practice to extract the LSP nucleon cross section from the the expected experimental event rates in order to compare with the SUSY predictions as a function of the LSP mass. In such analysis the factor t is omitted. It is clear, however, that, in going from the data to the cross section, one should divide by t. The results will be greatly affected for large reduced mass.

11 The Total Directional Event Rates

We will again examine separately the case of caustic rings and the isothermal models considered above.

11.1 The Total Directional Event Rates in the Case of Caustic Rings

Integrating Eqs. (102) - (105) we obtain:

1. In the first quadrant (azimuthal angle $0 \leq \phi \leq \pi/2$).

$$\begin{aligned} R^i_{dir} &= \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}[(r^i_z - \cos\alpha \; h^i_1)|\mathbf{e}_z.\mathbf{e}| \\ &+ (r^i_y + cos\alpha h^i_2 + \frac{h^i_c}{2}(|cos\alpha| + cos\alpha))|\mathbf{e}_y.\mathbf{e}| \\ &+ (r^i_x - sin\alpha h^i_3 + \frac{h^i_s}{2}(|sin\alpha| - sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \end{aligned} \tag{120}$$

2. In the second quadrant (azimuthal angle $\pi/2 \leq \phi \leq \pi$)

$$\begin{aligned} R^i_{dir} &= \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}[(r^i_z - \cos\alpha \; h^i_1)|\mathbf{e}_z.\mathbf{e}| \\ &+ (r^i_y + cos\alpha h^i_2(u) + \frac{h^i_c}{2}(|cos\alpha| - cos\alpha))|\mathbf{e}_y.\mathbf{e}| \\ &+ (r^i_x + sin\alpha h^i_3 + \frac{h^i_s}{2}(|sin\alpha| + sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \end{aligned} \tag{121}$$

3. In the third quadrant (azimuthal angle $\pi \leq \phi \leq 3\pi/2$).

$$\begin{aligned} R^i_{dir} &= \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}[(r^i_z - \cos\alpha \; h^i_1)|\mathbf{e}_z.\mathbf{e}| \\ &+ (r^i_y - cos\alpha h^i_2(u) + \frac{h^i_c(u)}{2}(|cos\alpha| - cos\alpha))|\mathbf{e}_y.\mathbf{e}| \\ &+ (r^i_x + sin\alpha H^i_3 + \frac{h^i_s}{2}(|sin\alpha| + sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \end{aligned} \tag{122}$$

4. In the fourth quadrant (azimuthal angle $3\pi/2 \leq \phi \leq 2\pi$)

$$\begin{aligned} R^i_{dir} &= \bar{R}\frac{2\bar{\rho}}{\rho(0)}\frac{t}{2\pi}[(r^i_z - \cos\alpha \; h^i_1)|\mathbf{e}_z.\mathbf{e}| \\ &+ (r^i_y - cos\alpha h^i_2 + \frac{h^i_c}{2}(|cos\alpha| - cos\alpha))|\mathbf{e}_y.\mathbf{e}| \\ &+ (r^i_x - sin\alpha h^i_3 + \frac{h^i_s}{2}(|sin\alpha| - sin\alpha))|\mathbf{e}_x.\mathbf{e}|] \end{aligned} \tag{123}$$

11.2 The Total Directional Event Rates in Isothermal Models

We remind the reader that in this case we take the difference of the rates in two opposite directions.

Integrating Eq. (99) we obtain

$$R_{dir} = \bar{R}[\rho^{'}(0)/(\rho(0)]\,(t^0/4\pi)\,|(1+h_1(a,Q_{min})cos\alpha)\mathbf{e}\,_z.\mathbf{e}$$
$$- \; h_2(a,Q_{min})\,cos\alpha\mathbf{e}\,_y.\mathbf{e} + h_3(a,Q_{min})\,sin\alpha\mathbf{e}\,_x.\mathbf{e}| \qquad (124)$$

note that in the above expressions, unlike the case of caustic rings, the rate is normalized to $t^0/2$ and not to t In other words the effect of folding with LSP velocity on the total rate is taken into account via the quantity t^0. All other SUSY parameters have been absorbed in $\bar{R}$, under the assumptions discussed above in the case of non-directional rates.

We see that the modulation of the directional total event rate can be described in terms of three parameters h_l, l=1,2,3. In the special case of $\lambda = 0$ we essentially have one parameter, namely h_1, since then we have $h_2 = 0.117$ and $h_3 = 0.135$.

Given the functions $h_l(a, Q_{min})$ one can plot the the expression in Eq. (124) as a function of the phase of the earth α.

12 Results and Discussion

The three basic ingredients of our calculation were the input SUSY parameters (see sect. 5), a quark model for the nucleon (see sect. 4) and the structure of the nuclei involved (see sect. 6). The input SUSY parameters used for the results presented in Tables 1,2 and 3 have been calculated in a phenomenologically allowed parameter space (cases #1, #2, #3 of Kane *et al* [21] and cases #4-9 of Castano *et al* [21]. Our own SUSY parameters will appear elsewhere [19].

For the coherent part (scalar and vector) we used realistic nuclear form factors and studied three nuclei, representatives of the light, medium and heavy nuclear isotopes (Ca, Ge and Pb). In Tables 8,9 and 10 we show the results obtained for three different quark models denoted by A (only quarks u and d) and B, C (heavy quarks in the nucleon). We see that the results vary substantially and are very sensitive to the presence of quarks other than u and d into the nucleon. The spin contribution, arising from the axial current, was computed in the case of a number of both light and heavy nuclei, including the ^{207}Pb system. For the isovector axial coupling the transition from the quark to the nucleon level is trivial (a factor of $g_A = 1.25$). For the isoscalar axial current we considered two possibilities depending on the portion of the nucleon spin which is attributed to the quarks, indicated by EMC and NQM. [13] The ground state wave function of ^{208}Pb was obtained by diagonalizing the nuclear Hamiltonian [46]-[48] in a 2h-1p space which is standard for this doubly magic nucleus. The momentum dependence of the matrix elements was taken into account and all relevant multipoles were retained (here only monopole and quadrupole).

In Table 5, we compare the spin matrix elements at $q = 0$ for the most popular targets considered for LSP detection ^{207}Pb, ^{73}Ge, ^{19}F,^{23}Na and ^{29}Si. We see that, even though the spin matrix elements Ω^2 in the case of ^{207}Pb are about factor of three smaller than those for ^{73}Ge obtained in Ref. [26] (see Table 5), their contribution to the total cross section is almost the same (see Table 6) for LSP masses around $100\,GeV$. Our final results for the quark models (A, B, C, NQM,

Table 8: The quantity $\langle dN/dt\rangle_0 = \bar{R}t$ in $y^{-1}Kg^{-1}$ and the modulation parameter h for the coherent vector and scalar contributions in the cases #1 - #3 and for three typical nuclei.

		Vector Contribution		Scalar Contribution			
		$\langle dN/dt\rangle_0$	h	$\langle dN/dt\rangle_0$			h
	Case	$(\times 10^{-3})$		Model A	Model B	Model C	
Pb	#1	0.264	0.029	0.151×10^{-3}	0.220	0.450	-0.002
	#2	0.162	0.039	0.410×10^{-1}	142.860	128.660	0.026
	#3	0.895	0.038	0.200×10^{-3}	0.377	0.602	-0.001
Ge	#1	0.151	0.043	0.779×10^{-4}	0.120	0.245	0.017
	#2	0.053	0.057	0.146×10^{-1}	51.724	46.580	0.041
	#3	0.481	0.045	0.101×10^{-3}	0.198	0.316	0.020
Ca	#1	0.079	0.053	0.340×10^{-4}	0.055	0.114	0.037
	#2	0.264	0.060	0.612×10^{-2}	22.271	20.056	0.048
	#3	0.241	0.053	0.435×10^{-4}	0.090	0.144	0.038

EMC) are presented in Tables 8, 9 for SUSY models #1-#3 of Kane *et al* [21] and Table 10 for SUSY models #4-#9 of Castano *et al* [21]

In discussing the effects of folding with the LSP velocity combined with the nuclear form factor and specialized our results for the target ^{127}I. To this end we considered only the scalar interaction and studied the effects of the detector energy cutoffs, by considering two typical cases $Q_{min} = 10,\ 20$ KeV.

Special attention was paid to the the directional rates and the modulation effect due to the annual motion of the earth.

We will start our discussion with the non isothermal velocity spectrum due to caustic rings resulting from the self- similar model of Sikivie et al [31].

The total rates are described in terms of the quantities t, r_x^i, r_y^i, r_z^i for the unmodulated amplitude and $h, h_1^i, h_2^i, h_3^i, h_c^i, h_s^i$ $i = u, d$ for the modulated one. In Table 11 I we show how these quantities vary with the detector energy cutoff and the LSP mass. Of the above list only the quantities t and h enter the non-directional rate. We notice that the usual modulation amplitude h is smaller than the one arising in isothermal models [17, 18]. The reason is that there are cancelations among the various rings, since some rings are characterized by $y_{nz} > 1$, while for some others $y_{nz} < 1$ (see Table 7). Such cancelations are less pronounced in the isothermal models. As expected, the parameter t, which contains the effect of the nuclear form factor and the LSP velocity dependence, decreases as the reduced mass increases.

In the case of isothermal models we will limit ourselves to the discussion of the directional rates. In the special case of the direction of observation being close to

Table 9: The spin contribution in the $LSP -^{207} Pb$ scattering for two cases: EMC data and NQM Model for solutions #1, #2, #3.

	EMC DATA		NQM MODEL	
Solution	$\langle dN/dt \rangle_0$ $(y^{-1}Kg^{-1})$	h	$\langle dN/dt \rangle_0$ $(y^{-1}Kg^{-1})$	h
#1	0.285×10^{-2}	0.014	0.137×10^{-2}	0.015
#2	0.041	0.046	0.384×10^{-2}	0.056
#3	0.012	0.016	0.764×10^{-2}	0.017

Table 10: The same quantities as in Table 8 in the case of Pb for the solutions #4 − #9. #8 snd #9 are no-scale models. The values of $\langle dN/dt \rangle_0 = \bar{R}t$ for Model A and the Vector part must be multiplied by $\times 10^{-2}$.

	Scalar Part				Vector Part		Spin Part		
	$\langle \frac{dN}{dt} \rangle_0$			h	$\langle \frac{dN}{dt} \rangle_0$	h	$\langle \frac{dN}{dt} \rangle_0$		h
Case	A	B	C				EMC	NQM	
#4	0.03	22.9	8.5	0.003	0.04	0.054	$0.80\, 10^{-3}$	$0.16\, 10^{-2}$	0.015
#5	0.46	1.8	1.4	-0.003	0.03	0.053	$0.37\, 10^{-3}$	$0.91\, 10^{-3}$	0.014
#6	0.16	5.7	4.8	0.007	0.11	0.057	$0.44\, 10^{-3}$	$0.11\, 10^{-2}$	0.033
#7	4.30	110.0	135.0	0.020	0.94	0.065	0.67	0.87	0.055
#8	2.90	73.1	79.8	0.020	0.40	0.065	0.22	0.35	0.055
#9	2.90	1.6	1.7	0.009	0.95	0.059	0.29	0.37	0.035

the coordinate axes the rate is described in terms of the three quantities t_0 and h_i, $i = 1, 2, 3$ (see Eq. (124)). These are shown in tables 12-14 for various values of Q_{min} and λ. For the differential rate the reader is referred to our previous work [17, 18]. We mention again that h_2 and h_3 are constant, 0.117 and 0.135 respectively, in the symmetric case. On the other hand h_1, h_2 and h_3 substantially increase in the presence of asymmetry.

13 Conclusions

In the present paper we have calculated the parameters, which described the event rates for direct detection of SUSY dark matter. We found that the event rates are quite small and only in a small segment of the allowed parameter space they are

Table 11: The quantities t and h entering the total non-directional rate in the case of the target ${}_{53}I^{127}$ for various LSP masses and Q_{min} in KeV. Also shown are the quantities r^i_j, h^i_j $i = u, d$ and $j = x, y, z, c, s$, entering the directional rate for no energy cutoff. For definitions see text.

		LSP mass in GeV						
Quantity	Q_{min}	10	30	50	80	100	125	250
t	0.0	1.451	1.072	0.751	0.477	0.379	0.303	0.173
h	0.0	0.022	0.023	0.024	0.025	0.026	0.026	0.026
r^u_z	0.0	0.726	0.737	0.747	0.757	0.760	0.761	0.761
r^u_y	0.0	0.246	0.231	0.219	0.211	0.209	0.208	0.208
r^u_x	0.0	0.335	0.351	0.366	0.377	0.380	0.381	0.381
h^u_z	0.0	0.026	0.027	0.028	0.029	0.029	0.030	0.030
h^u_y	0.0	0.021	0.021	0.020	0.020	0.019	0.019	0.019
h^u_x	0.0	0.041	0.044	0.046	0.048	0.048	0.049	0.049
h^u_c	0.0	0.036	0.038	0.040	0.041	0.042	0.042	0.042
h^u_s	0.0	0.036	0.024	0.024	0.023	0.023	0.022	0.022
r^d_z	0.0	0.274	0.263	0.253	0.243	0.240	0.239	0.239
r^d_y	0.0	0.019	0.011	0.008	0.007	0.007	0.007	0.007
r^d_x	0.0	0.245	0.243	0.236	0.227	0.225	0.223	0.223
h^d_z	0.0	0.004	0.004	0.004	0.004	0.004	0.004	0.004
h^d_y	0.0	0.001	0.000	0.000	0.000	0.000	0.000	0.000
h^d_x	0.0	0.022	0.021	0.021	0.020	0.020	0.020	0.020
h^d_c	0.0	0.019	0.018	0.018	0.017	0.017	0.017	0.017
h^d_s	0.0	0.001	0.001	0.000	0.000	0.000	0.000	0.000
t	10.0	0.000	0.226	0.356	0.265	0.224	0.172	0.098
h	10.0	0.000	0.013	0.023	0.025	0.025	0.026	0.026
t	20.0	0.000	0.013	0.126	0.139	0.116	0.095	0.054
h	20.0	0.000	0.005	0.017	0.024	0.025	0.026	0.026

above the present experimental goals. One, therefore, is looking for characteristic signatures, which will aid the experimentalists in reducing background. These are two: a) The non directional event rates, which are correlated with the motion of the Earth (modulation effect) and b) The directional event rates, which are correlated with both the velocity of the sun, around the center of the Galaxy, and the velocity of the Earth. We separated our discussion into two parts. The first deals with the elementary aspects, SUSY parameters and nucleon structure, and is given in terms of the nucleon cross-section. The second deals with the transition from the nucleon to the nuclear level. In the second step we also studied the dependence of the rates on the energy cut off imposed by the detector. We presented our results in a fashion understandable by the experimentalists. We specialized our results in the case of the coherent process in the case of ${}^{127}I$, but we expect the conclusions to be quite general.

The needed local density and velocity spectrum of the LSP were obtained in

Table 12: The quantities t^0, h_1 and h_m for $\lambda = 0$ in the case of the target ${}_{53}I^{127}$ for various LSP masses and Q_{min} in KeV (for definitions see text). Only the scalar contribution is considered. Note that in this case h_2 and h_3 are constants equal to 0.117 and 0.135 respectively.

		LSP mass in GeV						
Quantity	Q_{min}	10	30	50	80	100	125	250
t^0	0.0	1.960	1.355	0.886	0.552	0.442	0.360	0.212
h_1	0.0	0.059	0.048	0.037	0.029	0.027	0.025	0.023
t^0	10.	0.000	0.365	0.383	0.280	0.233	0.194	0.119
h_1	10.	0.000	0.086	0.054	0.038	0.033	0.030	0.025
t^0	20.	0.000	0.080	0.153	0.136	0.11	0.102	0.065
h_1	20.	0.000	0.123	0.073	0.048	0.041	0.036	0.028

two special classes: 1) Non isothermal models and 2) Isothermal models. As we have already mentioned the actual situation may be a combination of an isothermal contribution and late infall of dark matter. In the present treatment we consider each of these distributions separately.

In the first case case we assumed a late in-fall of dark matter into our galaxy. the needed parameters were taken from the work of Sikivie et al [31] in the context of a self-similar model, which yields 40 caustic rings. Our results, in particular the parameters t, see Table 11, indicate that for large reduced mass, the kinematical advantage of μ_r (see Eqs. (51)- (54) is partly lost when the nuclear form factor and the convolution with the velocity distribution are taken into account. Also, if one attempts to extract the LSP-nucleon cross section from the data, in order to compare with the predictions of SUSY models, one must take t into account, since for large reduced mass t is different from unity.

In the case of the non-directional total event rates we find that the maximum no longer occurs around June 2nd, but about six months later. The difference between the maximum and the minimum is about 4%, smaller than that predicted by the symmetric isothermal models [17, 18]. In the case of the directional rate we found that the rates depend on the direction of observation. The biggest rates are obtained, If the observation is made close to the direction of the sun's motion. The directional rates are suppressed compared to the usual non-directional rates by the factor $f_{red} = \kappa/(2\pi)$. We find that $\kappa = r_z^u \simeq 0.7$, if the observation is made in the sun's direction of motion, while $\kappa \simeq 0.3$ in the opposite direction. The modulation is a bit larger than in the non-directional case, but the largest value, 8%, is not obtained along the sun's direction of motion, but in the x-direction (galactocentric direction).

In the case of the isothermal models we restricted our discussion to the directional event rates. The reduction factor of the total directional rate, along the sun's direction of motion, compared to the total non directional rate depends, of course,

Table 13: The same as in the previous table, but for the value of the asymmetry parameter $\lambda = 0.5$.

		LSP mass in GeV						
Quantity	Q_{min}	10	30	50	80	100	125	250
t^0	0.0	2.309	1.682	1.153	0.737	0.595	0.485	0.288
h_1	0.0	0.138	0.128	0.117	0.108	0.105	0.103	0.100
h_2	0.0	0.139	0.137	0.135	0.133	0.133	0.133	0.132
h_3	0.0	0.175	0.171	0.167	0.165	0.163	0.162	0.162
t^0	10.	0.000	0.376	0.468	0.365	0.308	0.259	0.160
h_1	10.	0.000	0.174	0.139	0.120	0.114	0.110	0.103
h_2	10.	0.000	0.145	0.138	0.135	0.134	0.134	0.133
h_3	10.	0.000	0.188	0.174	0.167	0.165	0.164	0.162
t^0	20.	0.000	0.063	0.170	0.171	0.153	0.134	0.087
h_1	20.	0.000	0.216	0.162	0.133	0.124	0.118	0.107
h_2	20.	0.000	0.155	0.143	0.137	0.136	0.135	0.133
h_3	20.	0.000	0.209	0.182	0.171	0.168	0.166	0.164

on the nuclear parameters, the reduced mass and the asymmetry parameter λ [18]. It is given by the parameter $f_{red} = t_0/(4\pi\ t) = \kappa/(2\pi)$. We find that κ is around 0.6 for no asymmetry and around 0.7 for maximum asymmetry ($\lambda = 1.0$). In other words it is not very different from the naively expected $f_{red} = 1/(2\pi)$, i.e. $\kappa = 1$. The modulation of the directional rate increases with the asymmetry parameter λ and it depends, of course, on the direction of observation. For $Q_{min} = 0$ it can reach values up to 23%. Values up to 35% are possible for large values of Q_{mon}, but they occur at the expense of the total number of counts. In all cases our results, in particular the parameters t, see Table 11, and t_0, see Tables 12-14, indicate that for large reduced mass, the kinematical advantage of μ_r (see Eqs. (51)- (54) is partly lost when the nuclear form factor and the convolution with the velocity distribution are taken into account. To be more precise, if one attempts to extract the LSP-nucleon cross section from the data, in order to compare with the predictions of SUSY models, one must take t into account, since for large reduced mass t is different from unity. *Acknowledgments:* Partial support by TMR No ERB FMAX-CT96-0090 of

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References

[1] E.W. Kolb and M.S. Turner, *The Early Universe*, Addison, Wesley, Redwood City, CA ,1990.
P.J.E. Peebles, *Principles of Physical Cosmology*, Princeton University Press, 1993. M.S. Turner, *Cosmology:going beyond the big bang*, *Physics World*, September 1996, p. 31.

Table 14: The same as in the previous, but for the value of the asymmetry parameter $\lambda = 1.0$.

		LSP mass in GeV						
Quantity	Q_{min}	10	30	50	80	100	125	250
t^0	0.0	2.429	1.825	1.290	0.837	0.678	0.554	0.330
h_1	0.0	0.192	0.182	0.170	0.159	0.156	0.154	0.150
h_2	0.0	0.146	0.144	0.141	0.139	0.139	0.138	0.138
h_3	0.0	0.232	0.222	0.211	0.204	0.202	0.200	0.198
t^0	10.	0.000	0.354	0.502	0.410	0.349	0.295	0.184
h_1	10.	0.000	0.241	0.197	0.174	0.167	0.162	0.154
h_2	10.	0.000	0.157	0.146	0.142	0.140	0.140	0.138
h_3	10.	0.000	0.273	0.231	0.213	0.208	0.205	0.200
t^0	20.	0.000	0.047	0.169	0.186	0.170	0.150	0.100
h_1	20.	0.000	0.297	0.226	0.190	0.179	0.172	0.159
h_2	20.	0.000	0.177	0.153	0.144	0.142	0.141	0.139
h_3	20.	0.000	0.349	0.256	0.224	0.216	0.211	0.203

[2] For a recent review see e.g. G. Jungman *et al.*,*Phys. Rep.* **267**, 195 (1996).

[3] G.F. Smoot et al., (COBE data), *Astrophys. J.* **396** (1992) L1.

[4] E. Gawiser and J. Silk,*Science* **280**, 1405 (1988); M.A.K. Gross, R.S. Somerville, J.R. Primack, J. Holtzman and A.A. Klypin, *Mon. Not. R. Astron. Soc.* **301**, 81 (1998).

[5] A.G. Riess *et al*, *Astron. J.* **116** (1998), 1009.

[6] R.S. Somerville, J.R. Primack and S.M. Faber, astro-ph/9806228; *Mon. Not. R. Astron. Soc.* (in press).

[7] Perlmutter, S. *et al* (1999) *Astrophys. J.* **517**,565; (1997) **483**,565 (*astro-ph*/9812133).
S. Perlmutter, M.S. Turner and M. White, *Phys. Rev. Let.* **83**, 670 (1999).

[8] *Cosmological parameters,astro-ph*/9904051; *Phys. Rep.* **333-334** (1990), 619.

[9] D.P. Bennett *et al.*, (MACHO collaboration), A binary lensing event toward the LMC: Observations and Dark Matter Implications, Proc. 5th Annual Maryland Conference, edited by S. Holt (1995);
C. Alcock *et al.*, (MACHO collaboration), *Phys. Rev. Lett.* **74** , 2967 (1995).

[10] R. Bernabei et al., INFN/AE-98/34, (1998); R. Bernabei et al., it Phys. Lett. **B 389**, 757 (1996).

[11] R. Bernabei et al., *Phys. Lett.* **B 424**, 195 (1998); **B 450**, 448 (1999).

[12] J.D. Vergados, *J. of Phys.* **G 22**, 253 (1996).

[13] T.S. Kosmas and J.D. Vergados, *Phys. Rev.* **D 55**, 1752 (1997).

[14] M. Drees and M.M. Nojiri, *Phys. Rev.* **D 48**, 3843 (1993); *Phys. Rev.* **D 47**, 4226 (1993).

[15] A. Djouadi and M. Drees , QCD Corrections to the Neutralino Nucleon Scattering, TUM-HEP-370-00;PM/00-14;
S. Dawson, *Nucl. Phys.* **B359**,283 (1991)
M. Spira it et al, *Nucl. Phys.* **B453**,17 (1995)

[16] M.W. Goodman and E. Witten,*Phys. Rev.* **D 31**, 3059 (1985);
K. Griest, *Phys. Rev. Lett* **61**, 666 (1988); Phys. Rev. **D 38**, 2357 (1988) ; **D 39**, 3802 (1989);
J. Ellis, and R.A. Flores, *Phys. Lett.* **B 263**, 259 (1991); *Phys. Lett* **B 300**, 175 (1993); *Nucl. Phys.* **B 400**, 25 (1993);
J. Ellis and L. Roszkowski, *Phys. Lett.* **B 283**, 252 (1992).

[17] J.D. Vergados, *Phys. Rev. Let.* **83**, 3597 (1999)

[18] J.D. Vergados, *Phys. Rev.* **D 62**,023519-1 (200); astro-ph/0001190

[19] M. Gomez and J.D. Vergados, Direct LSP-Nucleus cross sections to Higgs exchange for large tanβ

[20] A. Bottino *et al.*, *Mod. Phys. Lett.* **A 7**, 733 (1992); *Phys. Lett.* **B 265**, 57 (1991); *Phys. Lett* **B 402**, 113 (1997); hep-ph / 9709222; hep-ph /9710296;
J. Edsjo and P Gondolo, *Phys. Rev.* **D 56**, 1789 (1997);
Z. Berezinsky *et al.*, *Astroparticle Phys.* **5**, 1 (1996);
V.A. Bednyakov, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko, *Phys. Lett.* **B 329**, 5 (1994).

[21] G.L. Kane *et al.*, *Phys. Rev.* **D 49**, 6173 (1994);
D.J. Castaño, E.J. Piard and P. Ramond, *Phys. Rev.* **D 49**, 4882 (1994);
D.J. Castaño, Private Communication;
A.H. Chamseddine, R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **49**, 970 (1982);
P. Nath, R. Arnowitt and A.H. Chamseddine, *Nucl. Phys.* **B 227**, 121 (1983);
R. Arnowitt and P. Nath, *Mod. Phys. Lett.* **10**, 1215 (1995); R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **74**, 4952 (1995); R. Arnowitt and P. Nath, *Phys. Rev.* **D 54**, 2394 (1996).

[22] R. Arnowitt and P. Nath, hep-ph/9701301; hep-ph/9902237;
S.K. Soni and H.A. Weldon, *Phys. Lett.* **B 126**, 215 (1983);
V.S. Kapunovsky and J. Louis, *Phys. Lett.* **B 306**, 268 (1993);
M. Drees, Phys. Lett **B 181**, 279 (1986);
P. Nath and R. Arnowitt, *Phys. Rev.* **D 39**, 279 (1989);
J.S. Hagelin and S. Kelly, Nucl. Phys. **B 342**, 95 (1990);
Y. Kamamura, H. Murayama and M. Yamaguchi, *Phys. Lett.* **B 324**, 52 (1994);
S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B 206**, 387 (1981).

[23] H.E. Haber and G.L. Kane, *Phys. Rep.* **117** 75 (1985).

[24] S.L. Addler, *Phys. Rev.* **D 11** (1975) 3309.

[25] T.P. Cheng, *Phys. Rev.* **D 38** 2869 (1988); H -Y. Cheng, Phys. Lett. **B 219** 347 (1989).

[26] M.T. Ressell *et al.*, *Phys. Rev.* **D 48**, 5519 (1993); M.T. Ressell and D.J. Dean, hep-ph/9702290.

[27] V.I. Dimitrov, J. Engel and S. Pittel, *Phys. Rev.* **D 51**, R291 (1995).

[28] J. Engel, *Phys. Lett.* **B 264**, 114 (1991).

[29] M.A. Nikolaev and H.V. Klapdor-Kleingrothaus, *Z. Phys.* **A 345**, 373 (1993); *Phys. Lett* **329 B**, 5 (1993); *Phys. Rev.* **D 50**, 7128 (1995).

[30] P.C. Divari, T.S. Kosmas, J.D. Vergados and L.D. Skouras, *Phys. Rev.* **C 61** (2000), 044612-1.

[31] P. Sikivie, I. Tkachev and Y. Wang, *Phys. Rev. Let.* **75**, 2911 (1995; *Phys. Rev.* **D 56**, 1863 (1997)
P. Sikivie, *Phys. Let.* **b 432**, 139 (1998); astro-ph/9810286

[32] A.K. Drukier et al., *Phys. Rev.* **D 33**, 3495 (1986);
J.I. Collar et al., *Phys. Lett* **B 275**, 181 (1992).

[33] P.F. Smith et al, Phys. Rep. **307**, 275 (1999); N. Spooner, *Phys.Rep.* **307**, 253 (1999); J.J. Quenby et al Astropart. Phys. **5**, 249 (1996); Phys. Lett. **B 351**, 70 (1995).

[34] J.R. Primack, D. Seckel and B. Sadoulet, Ann. Rev. Nucl. Part. Sci. **38**, 751 (1988).

[35] P.F. Smith and J.D. Lewin, Phys. Rep. **187**, 203 (1990).

[36] J.D. Vergados and T.S. Kosmas, *Physics of Atomic nuclei*, Vol. **61**, No 7, 1066 (1998) (from *Yadernaya Fisika*, Vol. 61, No 7, 1166 (1998).

[37] D. Spergel, *Phys. Rev.* **D 37**, 299 (1996).

[38] K.N. Buckland, M.J. Lehner, G.E. Masek, in *Proc. 1st Int. Workshop on IDM*, p. 475, Ed. N.J.C. Spooner, World Scientific, (1997); Proc. *3nd Int. Conf. on Dark Matter in Astro- and part. Phys.* (Dark2000), Ed. H.V. Klapdor-Kleingrothaus, Springer Verlag (2000).

[39] J.D. Vergados, *Phys. Rev.* **D 58**, 103001-1 (1998).

[40] J. Ashman et al., EMC collaboration, Nucl. Phys. **B 328** (1989) 1.

[41] R.L. Jaffe and A. Manohar, Nucl. Phys. **B 337** (1990) 509;
J. Ellis and M. Carliuer, The Strange Spin of the nucleon, CERN-Th/95334, Hep-ph 960 1280.

[42] P.M. Gensini, In Search of the Quark Spins in the Nucleon: A next-to-Next-to Leading Order in QCD Analysis of the Ellis-Jaffe Sum Rule, preprint DFUPG-95-GEN-01, hep-ph/9512440.

[43] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. **B 257** (1991) 83.

[44] H.E. Haber and R. Hempfling, Phys. Rev. **Lett. 66** (1991) 1815. Phys. Rev. **D 51** (1995) R291. i

[45] Copi, C.J., Heo, J. and Krauss, L.M. (1999),

[46] T.S. Kosmas and J.D. Vergados, Nucl. Phys. **A 510** (1990) 641;**A 536** (1992) 72.

[47] J.D. Vergados, Phys. Lett. **B 36** (1971) 12; **34 B** (1971) 121 .

[48] T.T.S. Kuo and G.E. Brown, Nucl. Phys. **85** (1966) 40.